Optimal Holding Company Organization and Capital Structure

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Abstract: Both the parent and subsidiaries of holding companies (HCs) can be financially leveraged. The subsidiaries may be simply bankruptcy-remote firewalled entities, created legally on paper and reported as off-balance sheet items, like Master Trusts and Conduits. Milti-tier HC capital structures have led to failures of many mega corporations like Enron and MCI-WorldCom. Major banks are now facing crises due to badly leveraged bankruptcy-remote entities. This paper shows how HCs should optimally determine their multi-tier financial leverage based on a novel debtholders' option to walk out (DOW) of bankruptcy proceedings should their out-of-pocket expenses exceed the value of recovery. We show that (a) DOW is too costly for many HCs to seek full diversification, for example, by mergers, and (b) DOW can optimally determine the number of tiers and the capital structure of a holding company.

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1 Introduction

This paper distinguishes a firm's organization by the number of tiers in its capital structure. A conglomerate is characterized by a one-tier capital structure for the entire firm including its subsidiaries. The conglomerate's debt is a claim on all its assets including those in its subsidiaries. A holding company (HC) is defined by a multi-tier capital structure, one for the parent and one for each subsidiary. The HC subsidiary's debt is a claim on only the assets of the subsidiary, while the parent's debt is a claim on assets of the parent. The HC parent's assets include equities in subsidiaries.[1]

1.1 Motivation

The real-world HCs often create subsidiaries which are simply bankruptcy-remote firewalled entities like Master Trusts and Conduits and report them as off-balance sheet items. Levered off-balance sheet entities have led to failures of many mega corporations like Enron and MCI-WorldCom. Multi-tier leverage is now hurting many major banks and riling the economy.

Reckless multi-tier leverage can adversely affect the economy and society. Consider, for example, a bank holding company (BHC) with 8 dollars of common equity and 92 dollars of insured deposits. The BHC can create a firewalled subsidiary with equity of 100 dollars infused from the parent bank. The subsidiary can then borrow 900 dollars of insured deposits. The parent BHC can thus meet the statutory minimum required capital-to-assets ratio of 8%.[2] The subsidiary's 10% capital-to-assets ratio exceeds the minimum regulatory capital requirement. On a consolidated basis, however, the BHC has only 8 dollars of equity for 1000 dollars of assets or 0.8% capital-to-assets ratio and is thus badly under-capitalized. The executives of this BHC may be tempted to bet the insured funds in the firewalled subsidiary on very risky investments. Such risky bets can generate high returns to the parent BHC's shareholders and executives, though with a small probability, but subject the insured deposits to the risk of losses to be borne by taxpayers. Such multi-tier leverage structures are currently grinding the U.S. banking industry. The taxpayers will be levied heavy losses due to bailing out of banks that are too big to fail.[3]

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[1] A real-world firm which has at least two tiers in its capital structure is called a Holding Company (HC) and is distinguished from an ordinary firm with a one-tier structure. A two-tier HC has the *first* tier of its capital structure at the level of each of its subsidiaries, and the *second* tier at the parent company level. The parent or each subsidiary is a separate legal *entity*, although the parent's stakeholders are jointly the equityholders of each subsidiary. The financial claims, e.g., debt and equity, of a particular entity are claims *only* against the assets of the entity. The assets of the parent entity include the equities of the subsidiaries. In addition, the parent HC can invest on its own assets. Then, the parent debt and equity are claims against these assets as well as the subsidiaries' equities. But no subsidiary's debtholders can stake their claims against these parent assets. A three-tier HC has one or more subsidiaries that are two-tier HCs, and so on. The existing literature does not distinguish firms based on the number of tiers of financial claims on their assets. The extant literature considers distinction among firms, based on the efficiency of management and operation.

[2] The minimum capital requirements are optimal from the point of view of taxpayers as shown in Acharya and Dreyfus (1989) and argued in Acharya (2008).

[3] During a meeting with the top management in a mega U.S. bank the author, on an assignment through the Federal Reserve Board, pointed out contravention of the bank capital requirement on a consolidated basis. The management was angry. The author has communicated with the U.S. Treasury Secretary about the necessity to impose the minimum bank capital requirement on a consolidated basis. The Treasury's idea of having a Super Investment Vehicle to buy up the assets from fire-walled BHC subsidiaries was then shelved and BHCs were cajoled to absorb the losses in such subsidiaries.

1.2 Debtholders' Option to Walkout of Bankruptcy

The above motivation makes it important to analyze and determine the organizational form and the leverage structure optimally. To do so, this paper introduces a novel idea: Debtholders have a put option to walk out (DOW) of a bankruptcy process, when the value of assets of the firm in the state of default is less than the out-of-pocket transaction cost needed to recover any value from the assets. If the firm defaults, debtholders can exercise their put option to not incur any out-of-pocket expenses. The real world debtholders have a limited liability by which they need not incur out-of-pocket expenses in the event of default. Since the lawyers handling a bankruptcy will continue their efforts as long as they are compensated, debtholders can ask their lawyers to stop the bankruptcy litigation when the assets of the firm are exhausted. The extant literature has not considered the debtholders' put option to walk out of bankruptcy proceedings.[4]

1.3 The Main Results

The paper then obtains the following results:

1. The value of DOW is crucial for optimal merger of independently operating firms into either a holding company or a conglomerate. Merging into a conglomerate can generate a smaller DOW value than the total of DOW values of the independent firms. A merger, however, can create an incremental positive value from diversification of risk of assets possible in a conglomerate. We show that it is suboptimal to merge independent firms with high DOW values into a conglomerate. The DOW value is high when the debtholders' out-of-pocket expense during bankruptcy is high. The incremental value from diversification of risk of assets of such independent firms does not compensate enough for the loss in the DOW value after conglomeration.

2. The HC structure is optimal for moderate DOW values and degree of diversification. The conglomerate is on one extreme end of the organizational form, whereas the independent firms are on the other. The HC structure is in between the two extremes. A multi-tier HC capital structure (as opposed to conglomerate) may be optimal for technological and industrial assets which impute large bankruptcy restructuring costs. Investing in such assets

[4] The debtholders' out-of-pocket expenses can be construed to include the financial intermediation cost as well as other bankruptcy/restructuring/legal fees, per dollar of lending, needed to transfer the assets of the firm from shareholders to debtholders in the state of default. The financial intermediation literature proves why banks should be formed: individual lenders face prohibitive costs of seeking and monitoring borrowers. Individual lenders benefit from their coalition (called bank) to diversify risk and reduce cost of lending. The diversification thus achieved is very different from that of a coalition of entrepreneurs or executives involved in forming HCs. Individuals lend a bank to fund the assets comprising the pool of loans made to borrowers. These borrowers may be the enterpreneurs of a holding company. The entrepreneurs borrow against the pool of assets of the parent HC or subsidiaries. Within our setting, debtholders could form a coalition (a financial intermediary) to reduce the cost of disclosing the true value of assets of the firm in the state of default like in Williamson (1986). Such a cost of financial intermediation could also be alternatively determined by a competition among several intermediaries in an economy (see, e.g., Ramakrishnan and Thakor (1984)). The financial intermediation literature is very rich. For other important works, see, e.g., Boyd and Prescott (1986), Chan (1983), Diamond (1984) and Gale and Hellwig (1984). Williamson shows on page 169 that the monitoring cost per dollar of lending (his c) will be shared equally by the depositors of the intermediary (his K), if the intermediary is allowed to monitor a large number of entrepreneurs (his $m \to \infty$). Then, Williamson argues on page 170 that the delegated monitoring cost will be driven down to zero for a large intermediary. [This argument is like Diamond's (1984) delegated monitoring cost $D_N \to 0$.] The monitoring cost of a fully diversified financial intermediary is, however, nontrivial equal to Williamson's $\frac{c}{K}$ per dollar of lending. This is because some cost is needed to ensure that each entrepreneur <u>discloses</u> his true payoff in the state of default. Ramakrishnan and Thakor (1984) argue that the financial intermediation cost could be determined competitively, when many intermediaries exist.

within separate subsidiaries of a holding company can generate higher values as selective exercise of DOWs is possible.[5]

3. The optimal capital structure of a holding company at each subsidiary and the parent is determined.

4. DOW is an optimal determinant of the number of tiers in the capital structure of a holding company. This result has important regulatory policy implications: The optimal leverage for a BHC derived within a deregulated environment can be the basis for the minimum bank capital requirement if regulators can be presumed to act as private surrogates of debtholders to impose debt covenants optimally. The private surrogate argument is the basis of determining the minimum capital requirement rule, optimally from the point of view of taxpayers, in Acharya and Dreyfus (1989). The optimally determined minimum threshold capital requirement has been statutorily enancted by governments around the world. Our results show that the minimum capital requirement should be enforced on a consolidated basis for a BHC, as opposed to separate enforcement for its each subsidiary and parent.

5. The degree of partial diversification of corporate firms is determined optimally. This is important because the real world does not form fully diversified conglomerates or HCs through mergers (portfolios) of independently operating firms, as implied by Marcowitz's (1952) seminal work on portfolio theory. Investors cannot generate the bankruptcy cost savings by forming their own portfolios of the independent firms. They will thus prefer fully diversified conglomerates formed by merger of independent firms. But most real world firms are not fully diversified conglomerates. This paradox is resolved by the novel idea of DOW.

6. There is an important positive *cost of diversification,* which is equal to the difference between the sum of DOW values of independent firms that can be merged into a conglomerate and the DOW value of such a conglomerate. The sum of the independent firm DOW values is shown to be strictly greater than the conglomerate DOW value, for a given total debt for either the conglomerate or independent firms.

   (a) The conventional wisdom does not entertain costs of diversification because the extant literature does not consider that debtholders can exercise their DOW.

   (b) The current wisdom rather involves a *benefit of diversification*, equal to the difference between the bankruptcy cost of a conglomerate and the total of bankruptcy costs for all of its subsidiaries if they were

[5] We show that the HC structure is beneficial because it can hold the assets within separate subsidiaries so as to regain the same DOW values which the subsidiaries would have as independent firms, instead of pooling the assets together like the conglomerate described earlier. The HC can also have another tier of debt at the parent company level and an associated DOW value, which can be traded against the cost of forming the extra tier to choose whether or not to have this tier. The HC structure can mimic the conglomerate by simply restricting its subsidiary debts to zero, which means that only the parent issues debt against the pooled assets. In another polar case, the HC can mimic the independent firms by issuing no parent debt, when it achieves no pooling of assets. More generally, the HC can *endogenously* determine a degree of diversification by pooling at the parent entity level some of the assets and the equities of the subsidiaries formed by the remaining assets. Our theory on the degree of diversification suggests that the real-world multi-tier capital structures will be based on assets (e.g., technological and industrial) which involve relatively large restructuring/bankruptcy costs, because holding such assets under separate subsidiaries has the advantage of selectively exercising the DOWs like in the case of the independent firms. HC structures are important not just for the traditional firms, but also for the ubiquitous bankruptcy-remote independent entities with assets funded by structured financings, which have swamped the financial market following the financial engineering innovations of the 1980s. For a good description of structured financings, see, e.g., *Credit Analysis of Structured Securities,* Moody's Investors Service, New York, 1991.

made independent firms. The diversification gain is always positive, although it approaches zero as the merger among independent firms result in a nearly perfectly diversified conglomerate.

(c) The diversification cost stemming from DOW is shown to be strictly positive even for a perfectly diversified conglomerate of independent firms. This leads to the result that leaving less than fully diversified firms as independent can be optimal.

(d) The diversification cost is shown to be an increasing function of the transaction cost in the state of bankruptcy. Casual empiricism suggests that the transaction cost in the state of default is relatively less for the financial firms than for the technological and the industrial firms. Our theory thus explains why the real world financial firms and intermediaries tend to be relatively better diversified than the technological or the industrial firms.

1.4 Values of Altertative Organizations

We compare the market values of three alternative investment strategies, distinguished by their organizational forms. Each strategy is based on the same number (m) of risky assets with correlated returns, but with no operating synergy. A lack of operating synergy is defined as: the expected payoff to any subset of the assets is equal to the sum of the expected payoffs to the individual assets in the subset.

- Investment Strategy $A1$: Own a holding company with some of the assets held by the parent entity, and with each of the remaining held by a subsidiary, and choose the capital structures of the parent and its subsidiaries.
- Investment Strategy $A2$: Own m independently operating firms, each with an asset and choose their capital structures.
- Investment Strategy $A3$: Own a conglomerate firm with all the assets and choose its capital structure.

In order to have a useful role of debt for a firm, we assume that each firm pays income taxes and/or managerial wages, both as fixed fractions of the *ex post* income after debt payments, if this income is positive, and no taxes or wages, otherwise. Higher debts result in lower managerial wages and lower income taxes, generating the benefits of debt. Each entity -- parent, subsidiary, independent firm or conglomerate -- incurs an entity formation cost, which does not vary with the *ex post* state of an entity, but increases with the number of entities of a firm. The entity formation cost includes (i) the borrowing cost, e.g., an underwriter fee if the debt is publicly raised or a lending fee if a bank loan is taken, and (ii) any state-independent managerial salary. Observe that the HC generates the same level of debt benefits due to a consolidation of its incomes across subsidiaries as the conglomerate -- although the set of independent firms are at a disadvantage. Thus, debt benefits are not crucial for our main results, they simply ensure an endogenous choice of debt within the model.

We compare the ‘‘pure’’ market values of the above three alternative investment strategies when the formation costs are equal to zero. We obtain the following results:

- $\mathcal{V}(A1)$ is greater than $\mathcal{V}(A2)$ as well as $\mathcal{V}(A3)$, where $\mathcal{V}(Aj)$ is the maximized market value of investment corresponding to the optimal capital structures in alternative Aj, $j = 1, 2, 3$.
- The value of a multi-tier capital structure, defined by $\mathcal{V}(A1) - \mathcal{V}(A2)$, is positive for all correlations of the asset returns, and is not due to the operating synergy among these firms.

- The result that $\mathcal{V}(A1)$ is greater than $\mathcal{V}(A3)$ appears very important at the time of a merger of an existing set of firms, because these firms can be organized either as a conglomerate with a one-tier capital structure or as an HC.

- $\mathcal{V}(A2) > \mathcal{V}(A3)$ for highly positively correlated asset returns and $\mathcal{V}(A2) < \mathcal{V}(A3)$ for highly negatively correlated asset returns.

1.5 Numerical Illustration

We then use numerical methods to examine the magnitudes of the values of the the three organizational forms (when the formation costs are zero), using two assets and reasonable exogenous parameter values: an income tax plus wage-incentive rate of 30 percent, a transaction cost (normal legal fee) of 10 percent of the face value of debt in the event of default, and lognormal distributions for the asset values. In particular, we determine (i) the capital structures and the maximized market values, and (ii) the value of converting an independently operating firm into a two-tier, one-subsidiary (''one-sub'') HC. The first set of numerical results show the extent of a financial structure-based value improvement feasible when two existing firms are merged into a two-sub, two-tier HC structure or into a conglomerate.

The first set of results are for nine different values of the correlation between the returns to assets of the two firms being merged. To abstract away from the size effect, we compare the maximized net present values (NPVs), which are net of the investments made. The results seem to be very interesting. For example, when the correlation is .4 (0), the conglomerate generates an NPV that is 22 (46) percent larger than the sum of the NPVs of the merging firms. Forming an HC, however, generates an NPV that is 78 (101) percent larger than the sum of the merging firms' NPVs. What seems to be very startling is that the NPV of the two-tier, two-sub HC is 72 (55) percent larger than the conglomerate NPV. We emphasize that these numerical results do not account for the formation costs, which will have to be considered along with the benefits of an HC or a conglomerate structure before choosing a merger of existing firms.

The second set of numerical results are for conversion of an existing firm, with a one-tier capital structure, into a one-sub HC for different values of the normal legal fee. These results show a substantial NPV improvement due to such a conversion. For example, for a normal legal fee of 10 percent of the face value of debt, the conversion results in 183 percent more NPV for the one-sub HC than for the independent firm. These results suggest that a multi-tier capital structure generates a higher value than a one-tier structure. Again, these numerical results do not account for the fixed formation cost as well as for the management personnel needed to run the parent entity created upon a conversion of an existing firm into a one-sub HC. For small firms, the improvement in the NPV due to a conversion may not be enough to justify the cost.[6]

The contractual terms of the parent and the subsidiary debt of a two-tier, one-sub HC can be argued to be similar, respectively, to the terms of junior and senior debt of an equivalent firm with a one-tier capital structure. Then, a corollary to our main result is that a junior-senior debt structure is more valuable than a capital structure with debt of the same seniority. Observe, however, that enforcing the corporate law for allocating the payoffs to debt and equity is likely to be much harder and costlier for a junior-senior debt structure than for a one-sub HC. Then the one-sub HC is likely to be preferred to a firm with a junior-senior debt structure. Most existing studies have demonstrated

[6] Evidence on such conversions of nonbanking firms is not easily available. But, bank data show that the average bank asset size of an independent banking firm is \$53 million, as opposed to \$116 million for a one-bank HC and \$2548 million for a multi-bank HC. [Bank Call Reports, 1985-1992, compiled by the Federal Financial Institutions Examination Council.

that debt is desirable in the capital structure of a firm facing some kind of market imperfection or taxes.[7] These studies do not, however, address why a firm should have varieties of debt identified by their seniority (as in the real world); and whether capital structure choices are relevant for forming a holding company (HC). In a very different model, Diamond (1992) shows that a manager could improve his benefits from controlling a firm by issuing debt of different maturities and by making the long-term debt junior to the short-term debt. Since we neither model benefits of control nor obtain a maturity structure for debt, our result implies that shareholders benefit from a senior-junior debt structure even when the debt is of the same maturity. In a recent, independent research, Winton (1992) considers a different model characterized by *multiple lenders* and shows that a firm could be better off borrowing debts of different seniority. Our model implies such a result, although we do not assume the existence of multiple lenders. To the best of my knowledge, none of the past studies focuses on the value of multi-tier, multi-sub HC structures, which cannot be mimicked by junior-senior debt structures.[8] Further, this paper seems to provide useful insights into the value of a multi-tier capital structure determined numerically.

We specify the model in Section 2, derive the capital structure and organizational choices in Sections 3 and 4, compute the value of alternative organizational structures by numerical methods in Section 5, and conclude in Section 6.

[7]See, e.g., Modigliani and Miller (1958, 1963), Scott (1976), Miller (1977), Myers (1977), Ross (1977), Leland and Pyle (1977), Haugen and Senbet (1978), Brennan and Schwartz (1978), Kim(1978), Chen and Kim (1979), DeAngelo and Masulis (1980), John and Nachman (1985), John (1987), and Harris and Raviv (1990), Acharya and Diwan (1993), among others.

[8]Works on financial intermediation that analyze holding company structures include Acharya (1991) and Kahn (1993); the latter (a very different model) is not based on optimal tiered capital structure considerations, which is the focus of the current paper. Garvey and Swan (1991) derive optimal capital structures for a hierarchical firm. As we discuss later, deriving an optimal capital structure for an HC (assuming that the HC structure exists) is very different from obtaining conditions under which an HC structure is valuable.

2 The Model

Consider an economy with m assets over two dates $t = 0, 1$, and three possible organizational forms, distinguished by their capital structures:

A1: a holding company (HC) with some of the assets owned by its parent entity and each of the remaining assets owned by a subsidiary,

A2: m independent firms, each owning an asset, and

A3: a conglomerate, which has no subsidiaries and which owns all the m assets.

We denote the parent of the HC by *entity* $e = p$, each of its potentially m subsidiaries by $e = 1, 2, ..., m$, the conglomerate by $e = c$, each independent firm by $e = 1, 2, ..., m$, and consider only debt and equity claims against the assets of any entity. Each entity is liquidated at time 1. In the case of an HC, each subsidiary's debt and equity constitute the *first-tier* of the capital structure, whereas the parent's debt and equity form the *second-tier* of the capital structure. Each independent firm or the conglomerate has only one-tier of debt and equity claims against their assets. In practice, HC structures may sometimes be called conglomerates. But, to distinguish firms based on the number of tiers of claims against their assets, we use the terminology *conglomerate* for a firm that results from a merger of the m assets, but adopts a one-tier capital structure. Our HC structure is not restricted to only the commonly recognized firms like the IBM. It applies as well to a structured (tiered) financing of a set of assets (e.g., loans and receivables from credit cards, autos, etc.) as in the asset backed securities market.

Firms are assumed to pay income taxes and managerial wages. The terminal payoff, before taxes and wages, of each independent firm and each subsidiary of the HC is denoted by $X_e,\ e = 1, ..., m$. The conglomerate's payoff before tax and wages is denoted by

$$X_c \equiv \sum_{e=1}^{m} X_e, \tag{1}$$

and the before-tax payoff of the parent entity is defined by

$$X_p \equiv \sum_{e=1}^{m} (X_e - F_e)^+, \tag{2}$$

where the amount of promised debt obligations of entity e is denoted by $F_e \geq 0$, $e = 1, 2, ..., m, c, p$; $(z)^+$ is equal to z if $z > 0$ and 0 otherwise. Choosing subsidiary e is equivalent to setting $F_e > 0$. Each independent firm or the conglomerate chooses its debt F_e and the HC chooses subsidiary debts, $(F_e,\ e = 1, 2, ..., m)$, as well as the parent debt, F_p. If $F_e = 0$ for all $e = 1, 2, ..., m$, the HC reduces itself to a conglomerate with a one-tier capital structure.[9]

Organizational choices are made on date 0, based on the market values. How exactly these market values are defined is not crucial for our results. We specify $X_e = X_e(1)$, $e = 1, 2, ..., m$, fairly generally.[10] We can thus

[9] In the real-world, some subsidiaries of an HC may have no debt. Further, the parent HC could be a guarantor of all the debt of other subsidiaries. Within our model, such subsidiaries are a part of the parent HC. Our model abstracts away from other instances in which a part of the debt of a subsidiary could be guaranteed by the parent.

[10] $\{X_e(u), 0 \leq u \leq 1\}$ can be specified to be generated by a vector of Brownian motions over continuous time in a filtered probability space $(\Omega, \mathcal{F}_{u(0 \leq u \leq 1)}, P)$, where Ω is the set of states, P the probability measure, and $\mathcal{F}_u$ the filtration at time u. Harrison and Pliska (1981, 1983) show that there exist an equivalent martingale pricing measure Q if and only the market is complete and arbitrage opportunities are precluded. Jarrow and Madan (1991) and Chatelain and Stricker (1994) give sufficient

determine the market values of the HC, the conglomerate, each ordinary firm, and their debt and equity claims, whose payoffs are functions of the payoffs to the assets.

We assume that the independent firms, the conglomerate and the HC maximize the market value of equity to choose their policies. The HC chooses its parent and subsidiary policies, consistent with the real world, where subsidiaries are not independent, although they are treated as separate entities. Thus, the parent and the subsidiary managements act in the interest of the parent shareholders subject to the corporate law that specifies how payoffs to assets be allocated to debt and equity. In essence, this law specifies that entity e's debt and equity are claims against only the assets of e. We assume that the manager of the HC, the conglomerate, or an independent firm is paid a wage *incentive* to maximize the market value of equity equal to a fraction $\iota \in (0,1)$ of the total income in excess of the debt obligations of the firm.[11] Given such a contract, we do not need income taxes for any of our results, because the incentive rate ι acts just like the income tax rate within our model, as made clearer later. Both income taxes and managerial wages strengthen our results. Any fixed wage paid to the manager would supplement an entity's formation cost introduced later.

For a capital-structure based organizational choice, debt must have some benefit, which in our model is implicit in the income taxes and the managerial incentive. Higher debts result in lower payments of taxes and incentives. We model for two sources of the cost of debt. First, debtholders of every entity e incur a bankruptcy cost, defined by

$$\underset{\text{Bankruptcy Cost}}{\gamma_e \equiv \gamma(X_e, F_e) \equiv \min(fF_e, X_e)J_e} = \underset{\text{Normal Legal Fee}}{fF_eJ_e} - \underset{\text{DOW Payoff}}{(fF_e - X_e)^+}, \tag{3}$$

where $f \in (0,1)$ is a constant and J_e is an indicator of the state of default of entity e, taking a value 1 whenever $X_e < F_e$ and $J_e = 0$, otherwise. Debtholders are assumed to have limited liability, i.e., bear no out-of-pocket expenses in the state of default and, hence, no legal fee if $X_e < fF_e$, while the lawyers handling the bankruptcy expend their resources as long as they are compensated, resulting in an effective bankruptcy cost of γ_e. The debtholders' option to walk (DOW) out of a bankruptcy process is valuable due to a nontrivial payoff equal to $(fF_e - X_e)^+$. As compared to a normal legal fee (specified in the extant literature), this DOW payoff is a valuable saving (legal fee avoidance). We assume that debtholders optimally exercise their put option when $fF_e > X_e$.

A proportional normal legal fee reflects the economy of scale and ensures a uniformity of the fee across the three alternatives if they were to have the same total debt. This also makes the HC's bankruptcy cost converge to the conglomerate's as the HC transfers all of its subsidiary debts to the parent. In the literature on financial intermediation, there is a cost per dollar of lending that even a fully diversified financial intermediary incurs to ensure that borrowers disclose their true payoffs in the states of default.[12] Diamond's (1984) maximum non-pecuniary penalty (to ensure

integrability conditions on the covariance matrix of the Brownian motions that generate $\{X_e(u)\}$ for the existence of the pricing measure and market completeness. We could specify a locally riskless discount factor process $\{\tilde{\beta}(u)\}$, with a spot interest rate process $\{r(u)\}$, i.e., $\tilde{\beta}(t) = \tilde{\beta}(0)Exp\left(\int_0^t r(u)du\right)$, with $\tilde{\beta}(0) = 1$. Cox, Ingersoll and Ross (1985) and Amin and Jarrow (1992) for locally riskless discount factor processes. Locally risky discount factors will result in market incompleteness (Acharya and Madan (1997)). If $y \equiv y(X_1(1), X_2(1), ..., X_m(1))$ is the random payoff at time 1 of a contingent claim on the assets and $\tilde{\beta}(1)$ is the discount factor over the period, then the market value of the claim at time 0 is $\mathcal{V}(y) \equiv E(\tilde{\beta}(1)y)$, where the expectation is evaluated under Q. With a little loss of generality, we assume for simplicity that $\tilde{\beta}(1) = Exp(-r) \equiv \beta$, where r is a constant instantaneous rate of interest.

[11] Indeed, it seems that the extant research could be extended to show that this wage contract is optimal, when shareholders (principal) maximize the market value of equity, for every given debt level, and the manager has an exponential utility of wealth (see, e.g., Holmstrom and Milgrom(1987) and Acharya (1992)).

[12] The notion of interest rate, in Williamson's (1986) models (3.6) and (4.5), rests on his specification that each of his K lenders lends \$1 and incurs a cost c is per dollar of lending. For a fully diversified intermediary, his monitoring cost is $\frac{c}{K}$ per dollar of

a full disclosure of the payoff) is F_e as opposed to our cost of fF_e in the state of default. Ramakrishan and Thakor (1984) argue that the cost of intermediation is competitively determined in a market with many intermediaries. Our per-dollar transaction cost f can be construed to include such costs for disclosure/intermediation, per-dollar of lending, as well as any other bankruptcy/restructuring/legal fees needed to transfer the assets of the firm from shareholders to debtholders. To check for the robustness of our results, we also consider a case in which every entity faces a fixed legal fee. The bankruptcy cost of a subsidiary e of the HC is equal to that of the independent firm, given debt F_e, $e = 1, 2, ..., m$. But, the parent entity of the HC also incurs an extra bankruptcy cost.

In the real world, firms also incur costs at the time of borrowing, which are independent of the *ex post* state of the firm. Even raising public debt involves initial underwriter fees, depending on the firm. We model this cost by an entity-specific, state-independent formation cost, equal to $\alpha_e I_e$, for $(e = 1, 2, ..., m, c, p)$, where α_e is a constant; $I_e = 1$ if and only if $F_e > 0$ and $I_e = 0$, otherwise, for $e = 1, 2, ..., m, c$; $I_p = 1$ if and only if $F_h > 0$ and $I_p = 0$, otherwise; and F_h is the total debt of the HC, given by

$$F_h \equiv \sum_{e=1}^{m} F_e + F_p. \tag{4}$$

The parent entity incurs the borrowing cost either when it borrows or when some of its subsidiaries borrow. The borrowing cost can also be alternatively interpreted as the cost of forming a levered entity, and α_p is the extra cost of forming the HC. Any fixed managerial wage is included in the formation cost, α_p, for the HC and α_e for the independent firms and the conglomerate; in this case, we interpret that the manager simply receives a fixed part of the terminal payoff of an unlevered firm.

We show that the debtholder bankruptcy cost is ultimately borne by the firm. By trading the bankruptcy and formation costs against the debt benefits, a one-tier entity e can determine its interior optimal capital structure. This is like the tradeoff between tax benefits and bankruptcy costs (e.g., Scott (1976) and Kim (1978)), or between managerial benefits and reorganization costs (Harris and Raviv (1990)). Indeed, our managerial incentives are similar to the managerial benefits in the Harris-Raviv model. We show that, if the fixed cost of formation α_e is zero, $e = 1, 2, ..., m, c, p$, then the market value of a multi-tier capital structure, $\mathcal{V}(A1) - \mathcal{V}(A2)$, is strictly positive, and that an HC structure is strictly preferred to a conglomerate, i.e., $\mathcal{V}(A1) - \mathcal{V}(A3) > 0$, where $\mathcal{V}(Aj)$ is the *maximized market value* of investment in alternative Aj, $j = 1, 2, 3$.

We also derive choices of organizational forms distinguished by the number of tiers in their capital structures, when α_e is nontrivial. We show that α_p must be sufficiently large for a nontrivial HC structure (with nonzero parent and subsidiary debts) to be less valuable than a conglomerate structure or the set of independent firms. Although our model is consistent with an *ex ante* asymmetry of information about asset payoff resolved via the costly state verification (e.g., Williamson (1986)) as well as imperfect information, it abstracts away from other informational asymmetries, e.g., those that give rise to adverse selection.

2.1 Payoff Allocations to Debt and Equity

The Internal Revenue Services (IRS) tax codes treat the entire liquidation payoff X_e as income and the entire debt repayment F_e as an allowable deduction of entity e.[13] We assume a flat-tax rate, $\bar{\tau} \in (0, 1)$, and no other deductions except the debt payments. Since taxes and incentives are paid whenever the taxable income is positive, we assume

lending.

[13]The nature of our results does not change if a part of X_e is treated as gross income and a part of F_e is allowed as a deduction.

that the tax rate and the incentive rate add up to $\tau \equiv \iota + \bar{\tau}$, for simplicity in exposition. Henceforth, we treat τ as a flat tax rate levied whenever the total income exceeds the debt obligations for all firms. The equityholders of the conglomerate or an independent firm receive a positive payoff allocation if and only if $X_e > F_e$, given by

$$S_e(X_e, F_e) = (1-\tau)(X_e - F_e)^+, \;\; e \in \{\text{independent firms, conglomerate}\}. \tag{5}$$

Debtholders of every independent firm, conglomerate or subsidiary receive F_e if $X_e > F_e$, and X_e, otherwise, after incurring the bankruptcy cost γ_e. Letting $B_e(X_e, F_e)$ denote this payoff,

$$B_e(X_e, F_e) = \min(X_e, F_e) - \gamma_e, \;\; e \neq p. \tag{6}$$

In practice, a parent HC pays the IRS all the income tax due from the entire HC. The parent HC, in turn, collects from each subsidiary taxes, which the subsidiary would have paid had it been an independently operating entity, i.e, $\bar{\tau}(X_e - F_e)^+$. Each subsidiary e thus pays its parent, $\bar{\tau}(X_e - F_e)^+$ in taxes and the remaining amount $(1-\bar{\tau})(X_e - F_e)^+$ as a *transfer,* i.e., remits a total of $(X_e - F_e)^+$ to its parent. This results in a gross income (payoff) to assets of the parent HC, equal to X_p, defined in (2). Tax codes do not require subsidiaries of an HC to pay taxes on the transfers (including dividend) to their parent HC. This does not, however, make the HC preferable to other firms with one-tier capital structures, because subsidiary dividends are included in the parent HC's taxable income. Thus, like an ordinary firm, the HC pays an income tax only once before paying dividends to parent stockholders. Under the IRS codes, however, an HC can choose to pay taxes either on its consolidated taxable income, or on entity incomes of the parent and each subsidiary. The consolidated taxable income of an HC is the sum of the pre-tax incomes minus the sum of deductible expenses of its subsidiaries and the parent, which is $(X_c - F_h)^+$, where F_h is the sum of debts in all subsidiaries of the HC, as defined in (4). The HC's entity income (incomes of the parent and subsidiaries) is equal $(X_p - F_p)^+$, because subsidiaries have no taxable income after their remittances to the parent HC. Observe that

$$\underbrace{X_c - F_h}_{\text{The HC's consolidated income}} \quad \leq \quad \underbrace{X_p - F_p,}_{\text{entity income}} \tag{7}$$

because

$$\begin{aligned} X_p - F_p &= \sum_{e=1}^{m}(X_e - F_e)^+ - F_p \\ &= \sum_{e=1}^{m}[X_e - F_e + (F_e - X_e)^+] - F_p \\ &= X_c - F_h + \sum_{e=1}^{m}(F_e - X_e)^+. \end{aligned} \tag{8}$$

Intuitively, the negative entity incomes can offset the positive entity incomes due to a consolidation. Although we presume that the HC pays taxes on its consolidated income, a multi-tier capital structure could be valuable even if the HC pays taxes on separate entity incomes, as examined later. The following lemma shows that the states of insolvency (default) of the parent entity, each independent firm or the conglomerate are the same if based on the before-tax payoff as if based on the after-tax payoff. The states of default of each subsidiary of the HC are unaffected by taxes.

Lemma 1: For each independent firm and the conglomerate,

$$\begin{aligned} X_e - F_e > 0 &\iff X_e - \tau(X_e - F_e)^+ - F_e > 0, \\ X_p - F_p > 0 &\iff X_p - \tau(X_c - F_h)^+ - F_p > 0. \end{aligned} \tag{9}$$

Proof: See the Appendix.

The allocations of the terminal payoff to equityholders, $S_p(X_p, F_p)$, and to debtholders, $B_p(X_p, F_p)$, of the parent HC are as follows:

$$S_p(X_p, F_p) = \left[X_p - \tau(X_c - F_h)^+ - F_p\right]^+, \quad (10)$$

$$B_p(X_p, F_p) = \min\left[F_p,\ X_p - \tau(X_c - F_h)^+\right] - \gamma_p. \quad (11)$$

2.2 Capital Structure Choice and Lognormal Payoff

We first show that each independent firm $(e = 1, 2, ..., m)$ will have a unique interior optimal capital structure, when the payoff is lognormal, as opposed to other distributions considered in the extant literature. We show that with a lognormal payoff, the value of the firm is not concave in its debt choice, F_e, but it is unimodal. We use a lognormal process primarily to exploit the rich asset pricing environment for ascertaining benchmark values of forming an HC structure by simulations presented later. The problem of choosing the leverage of an independent firm or the conglomerate $(e = 1, 2, ..., m, c)$ is straightforward:

$$\underset{\{F_e \geq 0\}}{\text{Maximize}} \quad V_e(F_e) \equiv E\{\beta S_e(X_e, F_e) - \alpha_e I_e + E\{\beta B_e(X_e, F_e)\}, \quad (12)$$

where $V_e(F_e)$ is the value of entity e. It can be checked that this value is equal to the entity's unlevered value plus the tax benefits minus the entity formation and bankruptcy costs:

$$\underset{\substack{\text{Levered}\\ \text{Value}}}{V_e(F_e)} = \underset{\substack{\text{Unlevered}\\ \text{Value}}}{(1-\tau)E[\beta X_e]} + \underset{\substack{\text{Value of Tax}\\ \text{Benefits}}}{\tau E[\beta \min(X_e, F_e)]} - \underset{\substack{\text{Formation}\\ \text{Cost}}}{I_e \alpha_e} - \underset{\substack{\text{Bankruptcy}\\ \text{Cost}}}{E[\beta \gamma_e]}, \quad (13)$$

for $e = 1, 2, ..., m, c$. The tax benefit in (13) is the value of the incremental cash flows due to a lower income tax in the presence of debt. In the state of solvency $\{X_e \geq F_e\}$ of an entity $(e = 1, 2, ..., m, c)$, taxes equal $\tau(X_e - F_e)$ if the entity is levered with a face value F_e, as compared to τX_e if it is not levered; resulting in an increase in the entity's cash flows equal to $\tau X_e - \tau(X_e - F_e) = \tau F_e$ due to leverage. In the complementary state $\{X_e < F_e\}$, the entity's taxes equal 0 if it is levered, as compared to τX_e if it is not levered; resulting in incremental cash flows of τX_e due to leverage. These two tax-related cash flows can be expressed as $\min(\tau F_e, \tau X_e)$. The following proposition shows the existence of an interior optimum leverage of an independent firm.[14]

Proposition 1: If $\alpha_e = 0$, there exists a unique interior optimal debt $\hat{F}_e$ for independent firm $e = 1, 2, ..., m$.

Proof: See the Appendix.

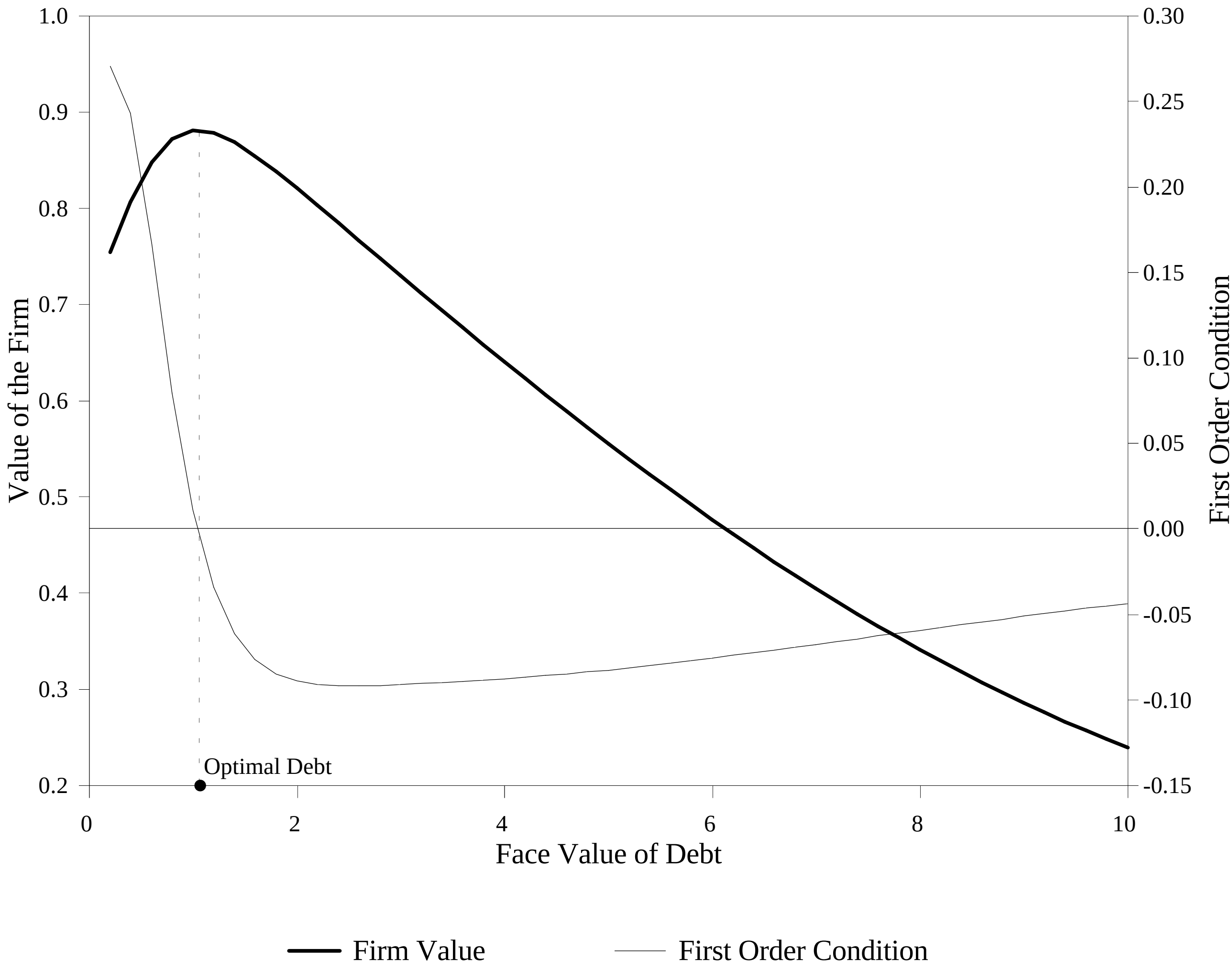
By Proposition 1, we can obtain the unique maximizer, $\hat{F}_e > 0$, by solving the first order condition of maximization of (13) for $\alpha_e = 0$ and $e = 1, 2, ..., m$. Figure 1 shows that the value of a firm is unimodal and concave at the maximum, as opposed to globally concave, and the first order condition for maximization has only one interior solution. The parameter values chosen to generate the plot are stated within the graph. The formula for the value of the firm, $V_e(F_e)$, is derived in equation (A5) and the first order condition, $\partial V_e / \partial F_e$, in (A8) of the Appendix. These two

[14] Proving such a result for the conglomerate appears difficult, and, therefore, a conglomerate's optimal capital structure choice is solved by numerical methods. This does not, however, affect our other results.

Figure 1

Optimal Capital Structure Choice Under Lognormal Payoff

The formula for the value of the firm, $V_e(F_e)$, is derived in equation (A5) and the first order condition, $\partial V_e/\partial F_e$, in (A8) of the Appendix. The parameters used to generate the graphs are, $f = .1, r = .1, \tau = .3, \sigma = .5$ and $X_e(0) = 1$.



are plotted against the amount of debt chosen by the firm. Observe, however, that an independent firm e will choose $\hat{F}_e$ as its optimal debt if and only if $V_e(\hat{F}_e) > V_e(0)$, which may not hold for a sufficiently large α_e. There exists, however, $\alpha_e > 0$, such that $V_e(\hat{F}_e) > V_e(0)$. This α_e satisfies

$$0 < \alpha_e < \tau E\{\beta \min(X_e, \hat{F}_e)\} - E\{\beta\gamma(X_e, \hat{F}_e)\}, \;\; e = 1, 2, ..., m, \tag{14}$$

when an independent firm will choose $\hat{F}_e > 0$ as its optimal debt.

3 Capital Structure and Organizational Choices

In this section, we derive the conditions under which one form of organization will be preferred over the other, i.e., compare the market values of investment in the alternatives $\mathcal{V}(Aj), \; j = 1, 2, 3$. Within our environment, wealth maximizing shareholders maximize the market value of their equity investment subject to the constraint that the market value of debt is equal to the amount of debt raised. The HC thus maximizes the market value of the parent equity to choose capital structures for the parent and subsidiaries. Since choosing a subsidiary is equivalent to setting $F_e > 0$, the HC solves for the face values of the subsidiary and the parent debts, $\{F \equiv (F_1, F_2, ..., F_m), F_p\}$; and for the corresponding amounts of debts to be raised, $\{D \equiv (D_1, ..., D_m), D_p\}$:

$$\begin{aligned} &\underset{\{F, D, F_p, D_p\}}{\text{Maximize}} \qquad W(F_p, F) \\ &\text{subject to} \qquad E\{\beta B_e(X_e, F_e)\} = D_e, \;\; e = 1, 2, ..., m, p, \end{aligned} \tag{15}$$

where

$$W(F_p, F) \equiv E\{\beta S_p(X_p, F_p)\} - \sum_{e=1}^{m} I_e\alpha_e - I_p\alpha_p + \sum_{e=1}^{m} D_e + D_p, \tag{16}$$

where $I_e = 1$ if and only if $F_e > 0$, and $I_e = 0$, otherwise, $e = 1, 2, ..., m, p$. The HC forms subsidiary e if and only if $D_e > 0$, i.e., $E\{\beta B_e(X_e, F_e)\} > 0$, when F_e is positive, since $F_e = 0$ implies that $\gamma_e = 0$ and hence $D_e = 0$. Thus, whenever subsidiary e is formed, the market value of its debt, D_e, is guaranteed to be positive and equal to the expected discounted payoffs to debtholders of the entity. Then, a positive amount of debt D_e can be successfully raised against assets of subsidiary e and debtholders of e can expect to realize a rate of return consistent with their default risk. The market value of parent debt D_p is likewise positive, whenever the HC levers the parent entity. The HC can be construed to raise enough equity from shareholders, initially, to fund its investments and then distribute the total debt raised, $D_p + \sum_{e=1}^{m} D_e$, as dividend to the parent shareholders. Thus, the maximand in (15) is the cum-dividend market value and the first expression in (16) is the ex-dividend market value of parent equity. This maximand implies that the objective of the HC shareholders is equivalent to maximization of the market value of the HC. The following lemma shows that the market value of the HC, $W(F_p, F)$, is equal to the unlevered value of the HC, plus the value of tax benefits, minus the value of the bankruptcy cost.

Lemma 3: The market value of the HC is given by

$$W(F_p, F) \equiv (1 - \tau)E[\beta X_c] + \tau E[\beta \min(F_h, X_c)] - E\left[\sum_{e=1}^{m} C_e + C_p\right], \tag{17}$$

where

$$C_e \;\; = \;\; I_e\alpha_e + \beta\gamma_e, \;\; e = 1, 2, ..., m, p. \tag{18}$$

Proof: See the Appendix.

In (17), the *ex post* tax benefits are $\tau\min(F_h, X_c)$. This is because in the state $\{F_h < X_c\}$, the HC's taxes equal $\tau(X_c - F_h)$ if it is levered with face values $(F_1, F_2, ..., F_m, F_p)$, as compared to τX_c if it is not levered; resulting in tax benefits of leverage, $\tau X_c - \tau(X_c - F_h) = \tau F_h$. In the complementary state $\{F_h \geq X_c\}$, the HC's taxes equal 0 if it is levered, as compared to τX_c if it is unlevered; resulting in tax benefits of leverage, τX_c. These two tax benefits can be expressed as $\min(\tau F_h, \tau X_c)$. Henceforth, we drop (D, D_p) as the choice variables of the HC, with the supposition that the market value of the HC is maximized to choose (F, F_p).

We can extend the cost-benefit tradeoffs of the previous section to an HC and a conglomerate and show (by numerical methods) that an interior optimal solution to (17)-(18) results in a higher value from leverage than the value from no leverage. That is, if $(\tilde{F}_p, \tilde{F} \equiv (\tilde{F}_1, \tilde{F}_2, ..., \tilde{F}_m))$ is an optimal solution to the (17)-(18), then $W(\tilde{F}_p, \tilde{F}) > \overline{W}$. The above capital structure analyses do not, however, result in an ordering of the maximized market values of investment in the three strategies, $\mathcal{V}(A1) = W(\tilde{F}_p, \tilde{F})$, $\mathcal{V}(A3) = V_c(\hat{F}_c)$ and $\mathcal{V}(A2) = \sum_{e=1}^{m} V_e(\hat{F}_e)$. The capital structure choices for an HC do not show why an HC structure exists (is desired) and hence provides no clue about whether a multi-tier capital structure is preferable to a one-tier capital structure.

3.1 Value of a Multi-tier Capital Structure

We first show that a two-tier capital structure (HC) is preferable to alternative $(A2)$ for some nontrivial $\alpha_p > 0$, whenever there exist $\alpha_e > 0$, $e = 1, 2, ..., m$, that satisfy (14), using two steps. In the first step, the following proposition asserts that the HC can fix each subsidiary's debt at a level that would be optimal if the subsidiary were an independent firm, and then choose an interior optimal parent debt for some nontrivial $\alpha_p > 0$. This is likely to be suboptimal, however, for the parent HC shareholders.

Proposition 2: Suppose that the HC sets the debt of its subsidiary e equal to the optimal debt of independent firm $e = 1, 2, ..., m$. Then, there exists $\alpha_p > 0$, such that the optimal debt of the parent entity, denoted by $F_p^*(\hat{F})$, given $\hat{F} \equiv (\hat{F}_1, \hat{F}_2, ..., \hat{F}_m)$, is strictly positive and

$$W(F_p^*(\hat{F}), \hat{F}) > W(0, \hat{F}). \tag{19}$$

Proof: See the Appendix.

Proposition 2 shows that it pays the HC to hold some subsidiary debt. In solving (15), given $F = \hat{F}$, there are m more constraints than in a standard leverage problem, namely, $E[\beta B_e(X_e, \hat{F}_e)] = D_e,\ e = 1, 2, ..., m$. These constraints, embedded in (17), guarantee that the subsidiary debtholders lend D_e for the given face value of debt $\hat{F}_e$ in subsidiary e. These constraints are satisfied under the supposition of Proposition 2 that $\hat{F}$ is given. Proposition 2 also shows that the parent HC, treated as a separate entity with a one-tier capital structure, will have a higher value when optimally levered than if it is all-equity financed. Intuitively, the parent entity trades off an increase in the market value of raising its debt from 0 to F_p against the increase in the expected discounted bankruptcy cost at the level of the parent company. This tradeoff results in an interior solution to the parent debt, given the subsidiary debts. Like in the case of the independent firms and the conglomerate, if α_p is sufficiently large, a levered parent entity will not be formed.

The second step in showing the desirability of a multi-tier capital structure is to compare the market value of investment in the independent firms (alternative $A2$) with the market value of investing in the HC structure (alternative $A1$). This is shown in the following proposition.

Proposition 3: The maximized market value of investment in alternative $A1$, determined by solving (15), $\mathcal{V}(A1)$, is strictly greater than the maximized market value of investment in $A2$, $\mathcal{V}(A2)$.

Proof: See the Appendix.

Proposition 3 does not depend on specific distributions of subsidiary payoffs, indicating that a positive value of a multi-tier capital structure does not stem from operating synergy considerations. The exact value of a multi-tier capital structure will, however, depend on the distribution of subsidiary asset returns, as shown numerically later. Intuitively, if the independent firms can choose optimal levels of debt, the HC can set its subsidiary debts equal to the optimal independent firm debts, respectively. Suppose first that the HC chooses no parent debt. Then the sum of the market values of the bankruptcy and formation costs of the independent firms will be equal to that of the subsidiaries of the HC. But, the HC structure can generate an equal or a greater market value of tax benefits due to a consolidation of incomes of the subsidiaries than that possible by the individual firms, even with no debt at the level of the parent as shown in (A20) in the Appendix. Observe, however, that Proposition 2 shows that for some nontrivial $\alpha_p > 0$, the parent entity can optimally choose some debt, resulting in a greater value of the HC when its parent is levered than if the parent is unlevered. The HC chooses some debt at the parent entity level only if the incremental market value of the tax benefits exceeds the increase in the market value of the bankruptcy and formation costs. This means that as long as the HC is able to optimally choose a positive parent debt, the result of Proposition 3 should not depend on how the HC pays its taxes, on its consolidated income or on the entity incomes. This is shown in the following proposition.

Proposition 4: Suppose that the HC pays taxes on entities' incomes, given by the right hand side of (7). Then there exists $\alpha_p > 0$, such that $\mathcal{V}(A1) > \mathcal{V}(A2)$.

Proof: See the Appendix.

3.2 The Choice between the HC and the Conglomerate

The next question we address is whether the market value of investment in alternative $A1$ is greater than that in $A3$. This is important because at the time of mergers of firms, either a conglomerate with a one-tier capital structure or an HC with a two-tier capital structure can be chosen. As discussed earlier, an HC structure with no subsidiaries ($F_e = 0,\ e = 1, 2, ..., m$) is identical to the conglomerate, when it is reasonable to presume that $\alpha_p = \alpha_c$. Thus $\mathcal{V}(A1) \geq \mathcal{V}(A3)$, where $\mathcal{V}(A3)$ is the maximized market value of the conglomerate firm. This does not, however, prove the optimality of a multi-tier capital structure, defined by an HC with at least one subsidiary and some parent debt.

Before proving the propositions in this section, we offer some intuition. Suppose first that the m assets are perfectly correlated. Then, comparing an HC formed by these assets, each held in a subsidiary, with a conglomerate formed by pooling the same assets is equivalent to comparing a one-sub HC formed by one of the assets with an independent firm (a trivial conglomerate) formed by the same asset. We can thus focus on a one-sub HC formed by, say, the first asset and compare it with an independent firm. Suppose that the independent firm's optimal debt is F_c, and that the one-sub HC restricts its total debt to F_c, but chooses F_s $(F_c > F_s > 0)$ for its only subsidiary and the remaining $F_p = F_c - F_s$ for its parent entity as in Figure 2. Suppose that the normal legal fee is a constant across all entities and is equal to $\hat{f} < F_e,\ e = c, s, p$. Figure 2 shows that the conglomerate has only one DOW, the tringular area, denoted DOW(s), with a payoff equal to $(\hat{f} - X_1)^+$, while the equivalent one-sub HC has an additional DOW(p), with a payoff $(\hat{f} - (X_1 - F_s)^+)^+$. The conglomerate (independent firm) can exercise its only DOW when $\hat{f} > X_1$. The HC can exercise one of its DOWs in the same set of states as the conlomerate, $\{\hat{f} > X_1\}$, and another DOW(p) when $F_s + \hat{f} > X_1 \geq F_s$. For both the HC and the conlomerate, the bankruptcy cost is equal to $\hat{f}$ in the states, $X_1 < F_c$, minus their DOW payoffs, implying that the HC's net cost is lower than that of the conglomerate.

The following proposition shows that a one-sub HC, even when it constrains its total debt to the level chosen by an equivalent independent firm (a trivial conglomerate), can strictly lower its expected bankruptcy cost as compared to the conglomerate's by splitting the debt between its subsidiary and the parent. This raises the value of an HC structure above the conglomerate's, because the tax benefits of under these two organizational forms are equal. Only when the one-sub HC holds all the debt at the parent or the subsidiary level, does its expected total bankruptcy cost become equal to the conglomerate's. The following proposition shows, however, that the one-sub HC will optimally transfer some debt to its subsidiary. This result does not depend on the proportionality of the normal legal fee, but on the DOW, as shown by the subsequent propositions in this section.

Proposition 5: Consider two alternative organizational structures based on the first of the m assets with payoff X_1: a conglomerate and a one-sub HC. Suppose that F_c is the conglomerate's optimal debt choice and the HC restricts its total debt (the parent's F_p plus the subsidiary's F_s) to F_c. (a) Then, there exists a positive subsidiary debt F_s $(0 < F_s < F_c)$ which generates $\mathcal{V}(A1|F_s + F_p = F_c) > \mathcal{V}(A3)$ if $\alpha_s = \alpha_c = \alpha_p = 0$; and (b) there exists $\alpha_e > 0,\ e = s, c, p$, such that $\mathcal{V}(A1|F_s + F_p = F_c) > \mathcal{V}(A3)$.

Proof: See the Appendix.

If the HC does not restrict its debt to that chosen by the conglomerate, it may achieve a higher value than that due to a reduction in the bankruptcy cost. In particular, by holding more debt than F_c, the HC may generate a higher tax benefit than the increase in the bankruptcy cost. Then $\mathcal{V}(A1) > \mathcal{V}(A3)$ for $\alpha_1 = \alpha_c = \alpha_p = 0$. In Figure 3, we plot the bankruptcy cost against the subsidiary debt of a one-sub HC, whose total debt is restricted to be equal to the optimal debt chosen by an equivalent conglomerate. The conglomerate's optimal debt, F_c, is chosen by solving the first order condition (A8) for maximization of the market value of firm 1. In this figure, the HC holds F_s in its subsidiary and $F_p = F_c - F_s$ at the parent entity level. As the subsidiary debt is increased, the bankruptcy cost of the subsidiary rises, while that of the parent falls. The figure shows that the total (parent plus subsidiary) cost is unimodal and the optimal subsidiary debt is uniquely determined. The HC achieves all the benefits of conglomeration, such as a potentially lower consolidated tax, and for optimal choices of its subsidiary and parent debts it can lower the bankruptcy cost below the conglomerate's. But, the HC bears a fixed formation cost for each entity, and the sum of these costs $\sum_{e=1}^{m} \alpha_e I_e + \alpha_p I_p$ could exceed the fixed formation cost of the conglomerate, α_c. Thus, there is a tradeoff between a potential reduction in the bankruptcy cost and the sum of the fixed formation costs in determining whether or not an HC should be formed, as opposed to a conglomerate.

The next proposition shows that the DOW is not necessary for the HC structure to dominate the conglomerate (when the formation costs are zero) as long as the legal fee is proportional to the face value of debt.

Figure 2

Debtholders' Option to Walk Out of a Bankruptcy Process in a one-sub HC.

The optimal conglomerate (independent) firm debt, F_c, is split between the subsidiary, F_s, and the parent F_p of the HC. The normal legal fee is equal to $\hat{f}$ for each entity. The HC exercises its DOW under two sets of states of payoff $\{F_s < X_1 < \hat{f} + F_s\}$ and $\{X_1 < \hat{f}\}$ as opposed to just the second set of states $\{X_1 < \hat{f}\}$ in which the conglomerate exercises its DOW.

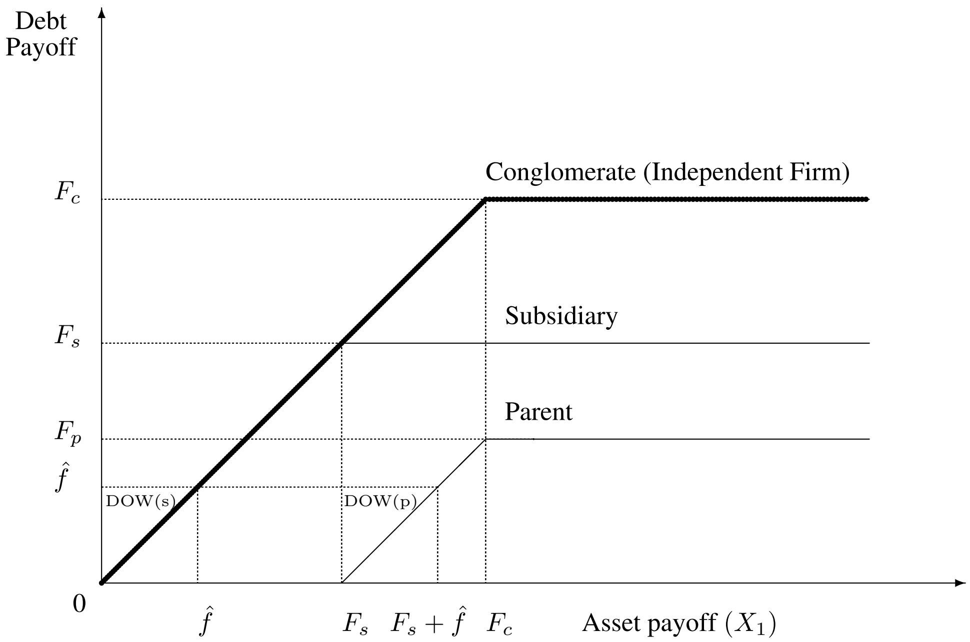
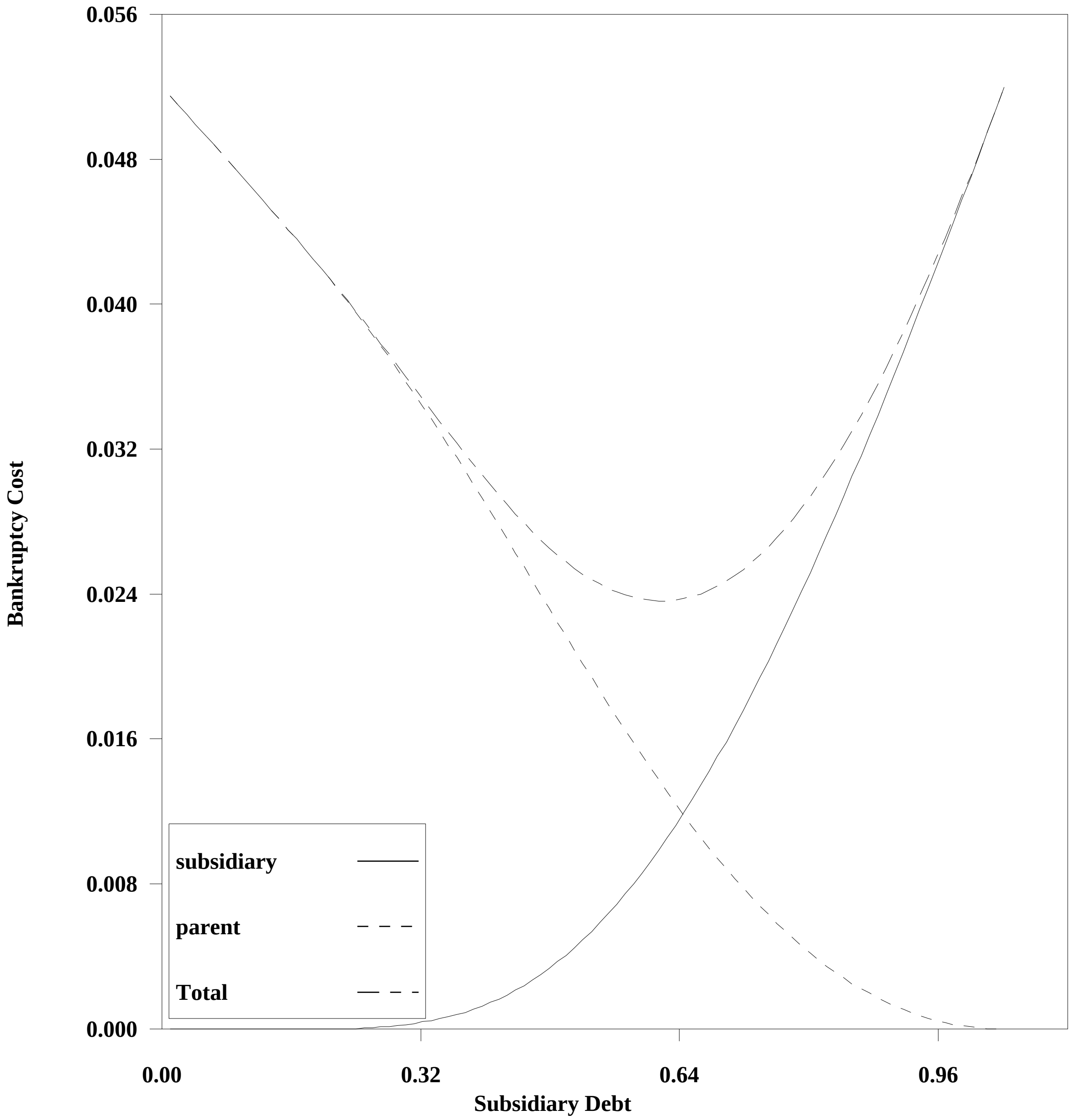


Figure 3

Choice Between an One-sub HC and a Conglomerate Structure

Both the one-sub HC and the conglomerate are based on only the first of the m assets. The value of tax benefits in either case is the same. The figure plots only the bankruptcy cost of the HC when it splits between its parent and subsidiary a restricted amount of debt equal to that optimally chosen by the conglomerate. The conglomerate's optimal debt is chosen by solving the first order condition (A8) for maximization of the market value of firm 1 in the Appendix. The conglomerate's bankruptcy cost corresponding to its optimum choice of debt is equal to that of the HC with a zero subsidiary debt or zero parent debt. We derive the formula for the expected value of the HC bankruptcy cost [sum of (A28) and (A29)] and the first order condition [sum of (A30) and (A31)] in the Appendix. The parameters used to generate the graphs are: $f = .1, r = .1, \tau = .3, \sigma_1 = .5$ and $X_1(0) = 1$.



Proposition 6: Suppose that the *ex post* bankruptcy cost of an entity is fF_eJ_e, and a one-sub HC restricts its total debt (the parent's F_p plus the subsidiary's F_s) to F_c, which is equal to an equivalent conglomerate's optimal debt choice. Then, (a) there exists a positive subsidiary debt F_s $(0 < F_s < F_c)$ which generates $\mathcal{V}(A1|F_s + F_p = F_c) > \mathcal{V}(A3)$ if $\alpha_s = \alpha_c = \alpha_p = 0$. (b) There exists $\alpha_e > 0,\ e = s, c, p$, such that $\mathcal{V}(A1|F_s + F_p = F_c) > \mathcal{V}(A3)$.

Proof: See the Appendix.

In Proposition 7, we show the desirability of an HC structure when the DOW is present, even when the legal fees are not proportional to the face value of debt and are equal across entities.

Proposition 7: Suppose that a one-sub HC restricts its total debt (the parent's F_p plus the subsidiary's F_s) to F_c, which is equal to an equivalent conglomerate's optimal debt choice and that the *ex post* bankruptcy cost of an entity is $f_eJ_e - (f_e - X_e)^+$, with $f_s = f_c = f_p$ and $f_e < F_e$. Then, there exists a positive subsidiary debt F_s $(0 < F_s < F_c)$ which generates $\mathcal{V}(A1|F_s + F_p = F_c) > \mathcal{V}(A3)$ if $\alpha_s = \alpha_c = \alpha_p = 0$, and (b) there exists $\alpha_e > 0,\ e = s, c, p$, such that $\mathcal{V}(A1|F_s + F_p = F_c) > \mathcal{V}(A3)$.

Proof: See the Appendix.

The following proposition shows that the DOW is crucial for the desirability of an HC structure if the legal fees are not proportional to the face values of debt, but are equal across the entities.

Proposition 8: Suppose that a one-sub HC restricts its total debt (the parent's F_p plus the subsidiary's F_s) to F_c, which is equal to an equivalent conglomerate's optimal debt choice and that the *ex post* bankruptcy cost of an entity is f_eJ_e, with $f_s = f_c = f_p$. Then, (a) the HC's optimal subsidiary debt F_s is zero and $\mathcal{V}(A1) = \mathcal{V}(A3)$ for $\alpha_s = \alpha_p = \alpha_c = 0$. (b) $\mathcal{V}(A1) \leq \mathcal{V}(A3)$ for $\alpha_s + \alpha_p \geq \alpha_c$.

Proof: See the Appendix.

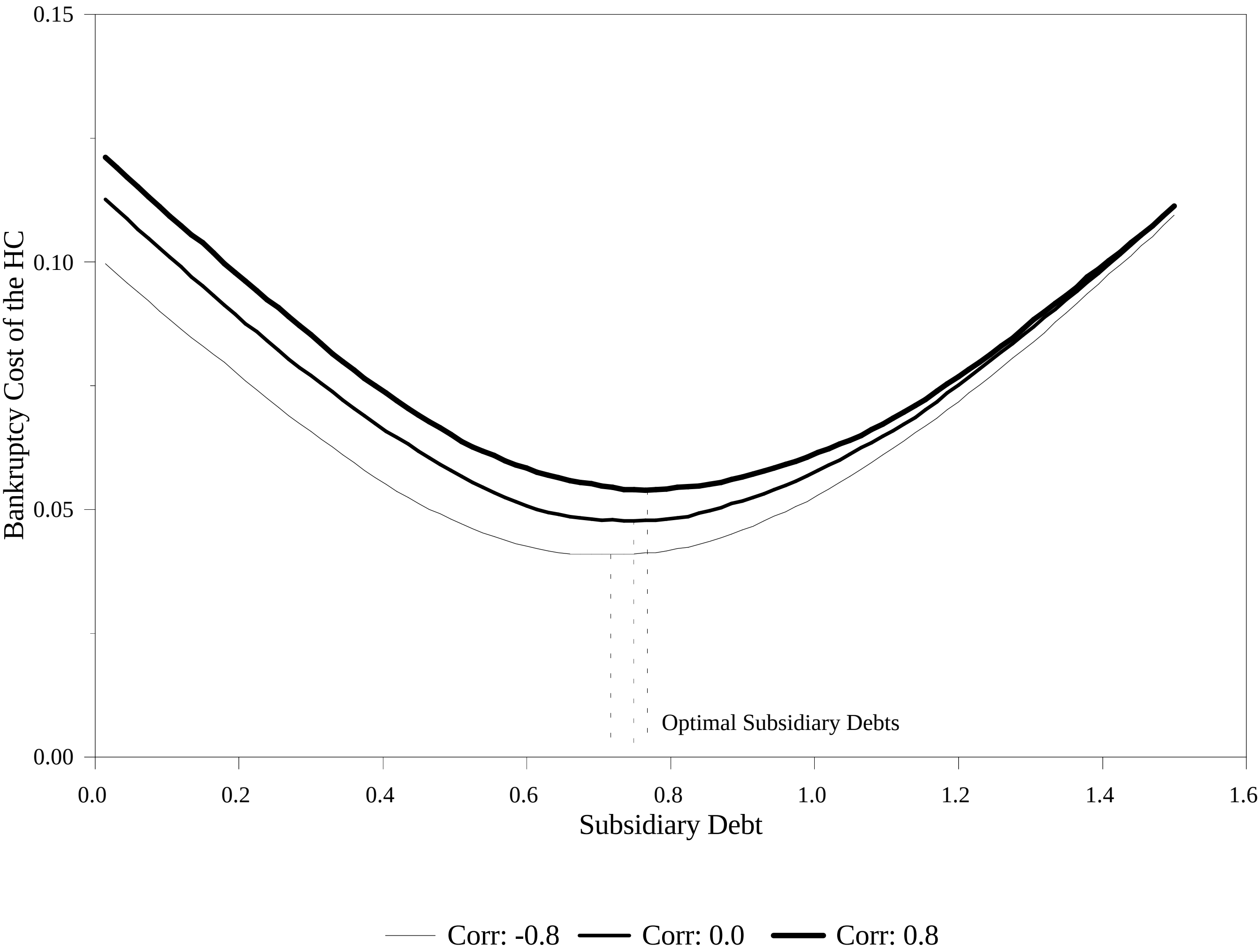
The HC has multiple entities as opposed to the single entity under a conglomerate structure. Since every entity has its own DOW, the HC can potentially exercise its DOWs more often than the conglomerate and, thereby, generate a higher value. We are not implying that the HC must necessarily appeal for bankruptcy whenever one of its subsidiaries defaults. Neither do we need such selective bankruptcies for our results to obtain. What is crucial is the restructuring of an entity and the option to not incur legal fees when there is very little recoverable from the assets of an entity. Subsidiary restructurings seem to be very difficult to document, because unless an entire HC becomes bankrupt, no record of a bankruptcy of the HC is likely to be available in the standard databases like in the Standard and Poor's Compustat. In fact, this data base documents all the HCs that file with the Security and Exchange Commission (SEC). During 1973-1992, we could document a total of 166 publicly traded parent holding companies with 245 subsidiaries (411 total number of entities), which filed their data with the SEC.[15] These are very large companies, such as AT&T, GE, Exxon, etc. Some of these holding companies have as many as 16 publicly traded subsidiaries. Out of the 245

[15] While Compustat includes about 150 banking firms, many of which are holding companies, the list of HCs that it records does not include any of the bank HCs. Compared to a total of 5756 bank holding companies out of a total of 9482 banking firms as of 1990 (Bank Call Reports), the recorded number of nonbank HCs appears to be a small number of the total of nonbank HCs in the Compustat data base.

Figure 4

Choice Between a Two-sub HC and a Conglomerate Structure

Either the two-sub HC or the conglomerate is based on the first two of the m assets. The value of tax benefits in either case is the same. The figure plots only the bankruptcy cost of the HC when it optimally splits the conglomerate's chosen debt, F_c, between its parent (F_p) and either subsidiary $(F_s = .5(F_c - F_p))$. The conglomerate's chosen debt maximizes its value. The conglomerate's bankruptcy cost corresponding to its optimum choice of debt is equal to that of the HC with a zero subsidiary debt. The value of the HC bankruptcy cost is computed by numerical methods. The common parameters used to generate the graphs are, $f = .1, r = .1, \tau = .3, \sigma_1 = \sigma_2 = .5, X_1(0) = 1$ and $X_2(0) = 1$.



subsidiaries, several were deleted during the period: 47 due to mergers, 2 due to bankruptcy, and 69 due to the reason that they were no longer required to file their information with the SEC. A firm (even a subsidiary) is required to file with the SEC only if it has some public debt or equity. Thus, the subsidiaries that stopped filing with the SEC were the ones that were restructured. How many of those merged were absorbed by the parent HC or other subsidiaries within the HC is difficult to identify. But, it appears that a lot of selective restructuring of the subsidiaries have taken place.

Since a theoretical comparison of the value of a conglomerate with that of a multi-sub HC appears difficult within the structure of our model, we use numerical methods and generate graphs similar to Figure 2 using two assets. The asset payoffs ($X_j,\ j = 1, 2$) are assumed to have the same marginal distribution, i.e., $ln(X_j) \sim N(\mu, \sigma^2)$ with $\mu = .1$ and $\sigma^2 = .25$. But, we consider three values of the correlation between the continuously compounded returns $ln(X_1)$ and $ln(X_2)$: $\rho = -.8,\ 0,\ .8$. We allow the HC to choose an equal amount of debt for either subsidiary, but restrict the total HC debt to be equal to the optimal conglomerate debt, F_c. Since the marginal distributions of X_1 and X_2 are the same, the restriction that either subsidiary have the same amount of debt may not be a real one. Our objective is to demonstrate that under these restrictions (especially, $F_p + 2F_s = F_c$), the HC can optimally choose $F_s > 0$ and attain a lower expected bankruptcy cost than the conglomerate's. We specify a risk-neutral economy and draw three random samples for the three correlations, each of size $N = 10000$, on $(ln(X_1^j), ln(X_2^j), j = 1, 2, ..., N)$ from a joint normal distribution with a mean vector $(.1, .1)$ and a variance vector $(.25, .25)$. The values of other parameters used for numerical estimation are $\tau = .3, f = .1, r = .1, X_1(0) = X_2(0) = 1$. We numerically estimate the F_c that maximizes the market value of the conglomerate, $V_c(F_c) \approx \beta \sum_{j=1}^{N} \left[\tau \min(X_1^j + X_2^j, F_c) - \min(X_1^j + X_2^j, fF_c) J_c^j\right]$, where $J_e^j = 1$ if and only if $X_e^j < F_e$ and $J_e^j = 0$, otherwise $(e = c, 1, 2, p)$. Using the estimated optimal F_c, we then compute the simulated value of the HC's bankruptcy cost as $\beta \sum_{j=1}^{j=N} \left[\min(X_1^j, fF_s) J_1^j + \min(X_2^j, fF_s) J_2^j\right]$ + $\beta \sum_{j=1}^{j=N} \min \left[(X_1^j - F_s)^+ + (X_2^j - F_s)^+, F_c - 2F_s)\right] J_p^j$ for different values of F_s. We plot the simulated HC bankruptcy cost against F_s in Figure 4 for each of the three samples. This figure shows that the subsidiary debt F_s that minimizes the HC bankruptcy cost is unique for each ρ. The higher the correlation between the returns to the assets, the larger the optimal subsidiary debt, under the restriction that $F_p = F_c - 2F_s$. The relationship of the correlation with the unrestricted optimal HC and conglomerate debts are discussed later.

Observe that the HC can mimic alternative (A2) by keeping the assets in separate subsidiaries, each with the same debt which the subsidiary would issue as an independent firm. The HC can also mimic alternative (A3) by issuing all its debt at the level of the parent. More importantly, the HC can optimally hold debt within its subsidiaries and the parent. In particular, by keeping the assets in separate subsidiaries and by choosing debts at the levels of subsidiaries as well as the parent, the HC can achieve a desired *degree of conglomeration*, which a straight conglomeration or the set of independent firms cannot accomplish. The choice of debt at two tiers allows the HC to determine how to pool the risk of the merging firms, whether through their assets or their equity via desired debt-equity mixes. The pooled subsidiary equity creates an extra tier of debt capacity against which the parent HC can issue debt. Propositions 5 through 8 assume that the HC pays a consolidated tax. But, the result of the next subsection shows that even with no consolidation of income, the HC can be preferred to a conglomerate structure if the assets are highly positively correlated, when the value of conglomeration is very low.

4 The Choice Between Independent Firms and the Conglomerate

In this section, we address the issue of diversification by entrepreneurs (shareholders), owning the independent firms. Because the independent firms do not have the benefits of consolidating their incomes, they generate a lower market value of the tax benefits than the conglomerate. Further, the diversification gain (when the DOWs are never exercised) of merging the independent firms into a conglomerate could be positive, presuming that debtholders would not exercise their put option to walk out of the bankruptcy process, when the transaction cost became greater than the recovery from assets. In this section, we show that when each independent firm's asset portfolio is nearly perfectly diversified, the diversification gain as well as the benefits of consolidation of income by the conglomerate will be nearly zero. But, the diversification cost defined by the difference between the sum of the independent firm DOW values and the conglomerate DOW value is strictly positive. This means that leaving the nearly perfectly diversified assets within the independent firms could be optimal depending on the formation costs.

We define the degree of diversification of an asset or a portfolio of assets by the variance of the rate of return to the assets, given the factors of an asset pricing model. Our results are not affected by whether the factors are specified by the Arbitrage Pricing Theory (Ross (1976)) or the Capital Asset Pricing Model (Sharpe (1964)). For the simplicity of the discussion, however, we consider the one-factor CAPM. Then

$$ln\left(\frac{X_e}{X_e(0)}\right) = a_e + b_e ln\left(\frac{X}{X(0)}\right) + \epsilon_e, \ \ e = 1, 2, ..., m, \tag{20}$$

where X is a lognormally distributed payoff to the market portfolio with a value $X(0)$ at time 0. The degree of diversification of the asset portfolio of the independent firm e is equal to the variance of ϵ_e, denoted by σ_e^2. Independent firm e's portfolio is fully diversified if $\sigma_e^2 = 0$. The larger the value of σ_e^2, the smaller the degree of diversification. The following proposition shows that there exist $\sigma_e^2 > 0$ and formation costs $\alpha_e > 0, \ e = 1, 2, ..., m$ which make the conglomerate an inferior choice to the independent firms.

Proposition 9: (a) There exist $\sigma_e^2 > 0$ and $\alpha_e > 0, \ e = 1, 2, ..., m$ with $\sum_{e=1}^{m} \alpha_e > \alpha_c$ such that, $\mathcal{V}(A3) < \mathcal{V}(A2)$. (b) Further, there exist $\alpha_e > 0$, $e = 1, 2, ..., m, c$, $\sum_{e=1}^{m} \alpha_e > \alpha_c$, such that $\mathcal{V}(A3) > \mathcal{V}(A2)$.

Proof: See the Appendix.

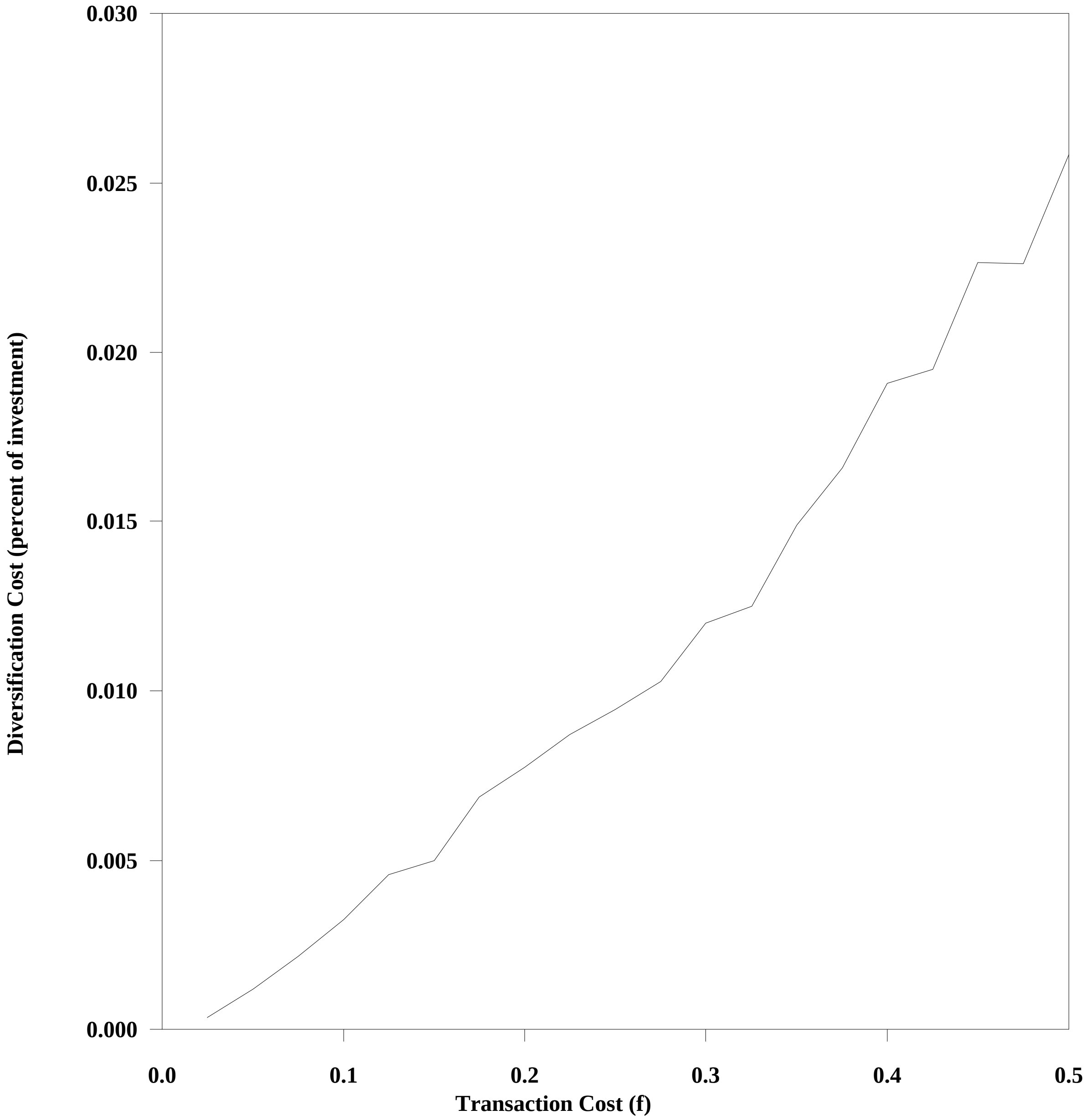
Intuitively, if the independent firm asset portfolios are fully diversified, the extra value of tax benefits from bringing the assets under a conglomerate structure is zero. As discussed earlier, the total bankruptcy cost can be split into two components: (i) the benefits of diversification equal to the transaction cost (legal fee) times the probability of bankruptcy and (ii) the DOW value. In the case of fully diversified independent firm assets, the diversification benefits of a conglomerate are equal to the sum of the diversification benefits of the independent firms. But, the sum of the independent firm DOW values is greater than the conglomerate DOW value. It is thus optimal to hold the fully diversified asset portfolios in a set of independent firms, assuming no formation cost. This result will also obtain, by continuity, for nearly fully diversified independent firm assets and some positive formation costs. For high enough costs of formation, however, the conglomerate will be preferred to the independent firms. The point that conglomeration can potentially reduce taxes, but not necessarily create an extra value to shareholders of merging firms is well known (see, e.g., Lam and Boudreaux, 1984). But, the extant literature neither analyzes the maximized market values under optimal capital structures, nor does it compare the values of conglomerates and HCs under optimal capital structures.

We then numerically compute the diversification cost of merging two independently operating firms. Recall that the diversification cost is the difference between the sum of the independent firm DOW values and the conglomerate DOW value. Figure 5 shows that the larger the transaction cost parameter f, the greater the diversification cost. This comparative statics and Proposition 9 imply, *ceteris paribus*, a lower degree of diversification of firms that should be

Figure 5

The Diversification Cost versus the Transaction Cost

The diversification cost is the difference between the sum of the two independent firm DOW values and the conglomerate DOW value. The conglomerate's chosen debt maximizes its value. Each independent firm is restricted to hold a half of this debt. The common parameters used to generate the graphs are, $r = .1, \tau = .3, \sigma_1 = \sigma_2 = .5, X_1(0) = 1$ and $X_2(0) = 1$ and zero correlation between the returns to the assets.



optimally left independent, the larger the transaction cost. That is, the larger the transaction cost of an independently operating firm, the greater the degree of diversification of the portfolio of assets of the firm.

Proposition 9 has an important implication for the regulatory policy on mergers of banks with highly diversified portfolios. Typically, large banks have highly diversified portfolios. It is very likely that a merger of such banks will reduce the value of DOWs which regulators exercise at the time of bank failures. Bank regulators have the right to exercise the DOW because of the bank deposit insurance. Proposition 9 offers an important caveat for allowing mega bank mergers, especially when they engender very little operating synergy.

5 Value of Alternative Organizational Structures by Numerical Methods

In this section, we use numerical methods to compare the maximized market values of multi-tier and one-tier capital structures, using 2 of the m assets. In the first subsection, we compute $\mathcal{V}(Aj), j = 1, 2, 3$. In the second subsection, we compare the value improvement due to conversion of an independently operating firm into a one-sub HC.

5.1 Two-sub, Two-tier Holding Companies

In this subsection, we form a two-sub, two-tier HC in strategy $A1$ by making two independently operating firms, each holding an asset, as the subsidiaries. We also form a one-tier conglomerate in $A3$ by merging the two firms. Strategy $A2$ is to leave the independent firms as they are. Further, there are two dates, $t = 0, 1$. We specify the payoff X_e to the assets of firm as a lognormally distributed random variable under the original measure:

$$ln(X_e) \sim N\left[\mu_e, \sigma_e^2\right], \;\; e = 1, 2. \tag{21}$$

To capture a possible correlation between X_1 and X_2, we specify:

$$ln(X_2) = a + b\, ln(X_1) + \epsilon, \tag{22}$$

where $b = \rho\sigma_2/\sigma_1$ and $a = \mu_2 - b\mu_1$ and ρ is the coefficient of correlation between $ln(X_1)$ and $ln(X_2)$.

We specify a risk-neutral economy and use a continuously compounded risk free interest rate, $r = .10$. The other parameters are specified as follows:

$$\begin{aligned}
\tau &= .30, \\
C_e &= \min(X_e, .1\, F_e),\; e = 1, 2, c, p, \\
\mu_e &= .10,\; e = 1, 2, \\
\beta &= e^{-r} = e^{-.10}, \\
\sigma_1^2 &= .25, \\
\sigma_2^2 &= .64.
\end{aligned}$$

A tax rate of $\tau = .3$ appears reasonable, so is the normal legal fee as a fraction of the face value of debt. The assumption on the normal legal fee subjects both the one-tier and two-tier structures to the same treatment. If the total HC's debt increases beyond the sum of debts of the independent firms, the HC will accordingly bear a greater cost of bankruptcy. A fixed fractional legal fee allows us to examine how the discounted expected (henceforth just expected) bankruptcy cost changes for different values of the correlation coefficient ρ.[16] We generate our numerical results for

[16] In the next subsection on the one-sub HC structure, we use different fractions in the definition of the maximum bankruptcy cost.

9 different values of ρ, from $-.8$ to .8. Different values of σ_1^2 and σ_2^2 did not change the nature of our results. But, the differences, $\mathcal{V}(A1) - \mathcal{V}(A2)$ and $\mathcal{V}(A3) - \mathcal{V}(A2)$, are sensitive to ρ. Using (21) and (22), we first generate $N = 1000$ random observations on X_1 and X_2 for each ρ. Increasing the number of observations does not change the results. Fewer observations ($N < 100$) appear to result in somewhat unstable results.

By denoting the n'th random observation by (X_{1n}, X_{2n}), we approximate the market values of debt and equity by the discounted mean of the payoffs to debt and equityholders evaluated at each of the n observations. We then solve the capital structure problems for the firms in strategy $A1$, $A2$ and $A3$, using RATS by Estima. Interior optimum capital structures yield in each case. For example, Figure 6 plots the NPV of investment in the HC against the parent entity debt for two cases, both with a value of correlation $\rho = .4$. In the first case, the HC chooses debt at the parent as well as the subsidiary level (reaching a value of E in Figure 6), whereas in the second case, the HC restricts the subsidiary debt to those amounts that the subsidiaries would have chosen had they been independent firms, generating a lower value A. The value of the all-equity parent HC is given by the horizontal dotted line passing through point B in Figure 6.

All the results, based on interior optimal capital structures as in Figure 6, are presented in Tables 1, 2 and 3. In Table 1, $v_e^{Aj} = \mathcal{V}(Aj)-$ (initial investment) is the NPV of investment in strategy $Aj,\ j = 1, 2, 3$ and entity $e = 1, 2, p, c$. We use the NPV measure to abstract away from the initial investments. The investment in assets of either independent firm, $e = 1, 2$, is \$1. Thus, the initial investment is \$2 in each strategy, $Aj,\ j = 1, 2, 3$. v_p^{A1} results from the optimal HC capital structure, denoted by face values of the subsidiary and the parent debt. Table 1 presents the market values of these optimal face values, denoted by D_e^{Aj}, for entity $e = 1, 2, p, c$ and strategy Aj, $j = 1, 2, 3$. Column 12 is $v_1^{A2} + v_2^{A2}$, which is the total NPV of investment in strategy $A2$. In Column 13, D_p^* is the market value of the parent HC debt corresponding to the optimal face value F_p^* of parent debt, which results when the subsidiaries' debts are restricted to be equal to the independent firms' optimal debt levels. Column 14 is the corresponding NPV of investment in an HC that follows such a constrained strategy. Column 15 is the market value of the optimal debt of the conglomerate and Column 16 is the NPV of investment in the conglomerate, i.e., in strategy $A3$.

Columns 12 and 16 show that $v_c^{A3} > v_1^{A2} + v_2^{A2}$, i.e., the strategy of forming a conglomerate structure by merging the two independent firms dominates the strategy of leaving the firms independent if the formation costs are zero. [For values of ρ, closer to 1 (not reported), this inequality is reversed.] The conventional wisdom is that for positive (negative) values of ρ, the ‘‘debt capacity’’ of the conglomerate should go down (up), because the combined assets become more (less) risky. Further, for $\rho = 0$, the debt capacity should not change in the absence of other synergies. Table 1, for $\rho = 0$, shows a higher conglomerate debt (\$1.83 in Column 15) than the debt capacity of the merging firms (\$1.65 in Column 9). Although the merging firms have no other synergy, the conglomerate realizes a higher benefit of debt due to a potentially lower consolidated tax, resulting in a larger amount of debt from the tradeoff against the costs associated with the bankruptcy. Column 15 shows that the conglomerate's debt capacity is lower (higher) than the \$1.83 benchmark debt capacity for positive (negative) correlations (except for $\rho = -.4$), consistent with the conventional wisdom.

Columns 6 and 12 in Table 1 show that $v_p^{A1} > v_1^{A2} + v_1^{A2}$, indicating that the HC dominates the strategy of leaving the two firms independent, i.e., that the value of a multi-tier capital structure $(\mathcal{V}(A1) - \mathcal{V}(A2))$ is positive if

Figure 6

The NPV of Investment in an HC, when the subsidiary asset return correlation is .4. The results are based on numerical estimation of the optimal capital structures.

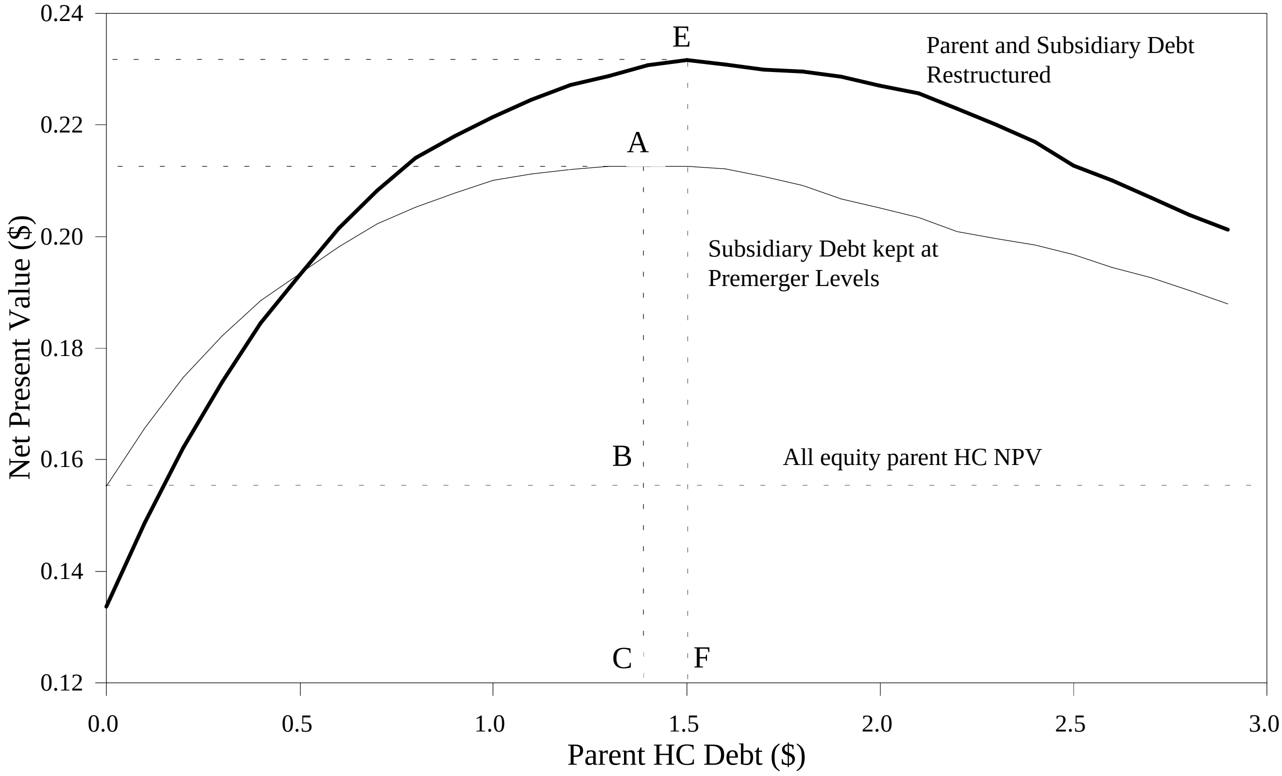


Table 1
Numerically Estimated Values ($) For a Two-sub, Two-tier HC and a Conglomerate

	HC					Independent Firms						HC*		Conglomerate	
ρ	D_1^{A1}	D_2^{A1}	D_p^{A1}	2+3+4	v_p^{A1}	D_1^{A2}	D_2^{A2}	7+8	v_1^{A2}	v_2^{A2}	10+11	D_p^*	v_p^{A1*}	D_c^{A3}	v_c^{A3}
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
-.8	.52	.60	.84	1.97	.31	.83	.87	1.70	.01	.15	.16	.27	.28	1.90	.27
-.6	.65	.62	.72	1.99	.28	.82	.85	1.66	-.01	.16	.14	.32	.26	1.85	.23
-.4	.66	.70	.63	1.99	.30	.76	.83	1.59	-.01	.18	.17	.43	.29	1.80	.24
-.2	.66	.71	.67	2.03	.32	.82	.84	1.66	-.01	.21	.20	.40	.31	1.88	.26
.0	.75	.61	.65	2.01	.23	.82	.83	1.65	.00	.11	.11	.39	.21	1.83	.17
.2	.66	.61	.72	1.99	.29	.82	.90	1.71	-.03	.20	.18	.33	.26	1.81	.22
.4	.75	.69	.53	1.97	.23	.81	.82	1.64	-.02	.15	.13	.38	.21	1.74	.16
.6	.76	.72	.56	2.04	.28	.83	.86	1.69	.01	.17	.18	.42	.27	1.76	.20
.8	.77	.70	.56	2.03	.24	.84	.83	1.68	.01	.14	.14	.39	.23	1.72	.15

*Subsidiaries' debts are restricted to be equal to the independent firms' optimal debt levels (D_1^{A2}, D_2^{A2}).

the formation costs are zero. Further, Columns 6 and 16 show that $v_p^{A1} > v_c^{A3}$, i.e., absent the formation costs, the HC dominates the conglomerate structure for all values of the correlation coefficient. The HC uniformly (for all ρ's) shifts its debt burden from the subsidiaries (as compared to their pre-merger optimal debts) to the parent entity. For example, when $\rho = .4$, the optimal independent firm debts are is .81 and .82, but the HC optimally lowers these to .75 and .69, respectively, after the firms become subsidiaries. The total debt capacity of the holding company is uniformly higher than the combined debt of the independent firms, even when $\rho = 0$. Further, for $\rho = 0$, the total optimal debt of the HC ($2.01) is higher than that of the conglomerate, indicating that by pooling equity, as opposed to pooling assets possible under a one-tier conglomeration, the HC is able to improve the debt capacity and the NPV.

The most striking point emerging from Table 1 is that the difference between the value of the conglomerate and that of the independent firms [Column (16) minus Column (12)] is decreasing in the correlation, as discussed following Proposition 9. On the contrary, the difference between the values of the HC and the conglomerate [Column (6) minus Column (16)] is increasing in the correlation. Further, the difference between the values of the independent firms and the HC [Column (6) minus Column (12)] does not vary much with respect to the correlation of the asset returns. While the disadvantage of the independent firms vis-a-vis the conglomerate merely declines with the correlation, the HC remains comparatively immune to changes in the values of correlations, because it can endogenously adjust its subsidiary and parent debt, and thereby achieve a desirable degree of risk that is not possible with the other two alternatives.

Table 2 shows the market value of the expected bankruptcy cost for each entity, equal to $E\{\beta \min(X_e, fF_e) J_e\}$. The subsidiary bankruptcy costs in Columns 2 and 3 are, respectively, less than the independent firms' costs in Columns 6 and 7. Even the total expected bankruptcy cost for the HC in Column 5 is not more than the total for the independent firms in Column 8, except for $\rho = -.4, .8$. Further, the value of the bankruptcy cost of the conglomerate

Table 2
Numerically Estimated Present Value of Bankruptcy Costs ($)

ρ	Two-sub, Two-tier HC				Independent Firms			Conglomerate
	Sub 1	Sub 2	Parent	Total	Firm 1	Firm 2	Total	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
-.8	.01	.03	.06	.09	.06	.10	.16	.09
-.6	.02	.02	.06	.10	.06	.08	.14	.09
-.4	.02	.04	.06	.12	.04	.08	.11	.10
-.2	.02	.04	.07	.13	.07	.08	.14	.14
.0	.04	.03	.07	.14	.06	.08	.14	.14
.2	.02	.02	.08	.12	.07	.09	.16	.14
.4	.04	.04	.06	.14	.06	.08	.14	.15
.6	.04	.04	.07	.14	.06	.07	.14	.14
.8	.04	.04	.08	.16	.06	.08	.14	.15

is comparable to that of the independent firms. We showed, theoretically, that for the same amount of debt held by the HC and the conglomerate, the bankruptcy cost of the latter will be less than that of the former. This does not mean that the expected bankruptcy costs would be necessarily different across these alternatives, when both the conglomerate and the HC choose debt, optimally. Indeed, due to a lower bankruptcy cost, the HC will increase its debt to take advantage of the extra tax benefits and achieve a higher value. We thus find no appreciable difference between the expected bankruptcy costs in columns (5) and (9).

Table 3 presents the improvement in the NPV of investment in the HC $(\mathcal{V}(A1)- \mathcal{V}(A2))$ in Column 2, which is equal to Column 6 $-$ Column 12 in Table 1. Likewise, the improvement in the NPV of investment in the conglomerate $(\mathcal{V}(A3) - \mathcal{V}(A1))$, presented in Column 6 of Table 3, which is equal to Column 16 $-$ Column 12 in Table 1. These improvements are expressed as percents of the sum of the NPVs of investment in the independent firms (Column 12 in Table 3). To examine the source of these substantial improvements in the NPV, we decompose the improvements into two parts: (i) due to the reduction in the expected bankruptcy cost, which is equal to Column 8 $-$ Column 5 (Table 2) for the HC and Column 8 $-$ Column 9 (Table 2) for the conglomerate, and (ii) the rest, which is due to the risk pooling and creation of the debt capacity.[17] These two components are reported as percentage NPV improvements in Table 3, Columns 4 and 5 for the HC and in Columns 8 and 9 for the conglomerate, showing that the expected bankruptcy cost the smaller of the two parts of the NPV improvement. The main source of the NPV improvement is asset risk pooling by the conglomerate, and equity risk pooling and the extra tier of capital structure in the HC.

Table 3 also shows the dollar and the percentage improvement in the NPV of investment in an HC structure as

[17] There are minor differences in the presented numbers due to rounding. The calculations are based on unrounded estimates.

Table 3
Sources of Improvement in Net Present Values

ρ	HC−Firms				Conglomerate−Firms				HC−Conglomerate			
	Improvement HC−Firms		Due to		Improvement Cong.−Firms		Due to		Improvement HC−Cong.		Due to	
			Bank. Cost Decr.	Eq Risk Pooling & Debt Cap			Bank. Cost Decr.	Ass Risk Pooling & Debt Cap			Bank. Cost Decr.	Eq Risk Pooling & Debt Cap
	(in $)	(in %)	(in %)	(in %)	(in $)	(in %)	(in %)	(in %)	(in $)	(in %)	(in %)	(in %)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
-.8	.15	92	48	52	.11	69	65	35	.04	25	0	100
-.6	.14	96	26	74	.09	64	54	46	.05	33	29	71
-.4	.13	76	-1	101	.07	44	25	75	.05	42	36	64
-.2	.13	66	10	90	.07	34	4	96	.06	48	-16	116
.0	.12	101	2	98	.05	46	-4	104	.06	55	-6	106
.2	.11	62	32	68	.04	24	43	57	.07	62	-25	125
.4	.10	78	1	99	.03	22	-14	114	.07	72	-7	107
.6	.10	55	-5	105	.02	10	-33	133	.08	82	-1	101
.8	.09	65	-20	120	.01	4	-167	267	.09	94	10	90

compared to that in a conglomerate structure in Columns 10 and 11, respectively. These results show that the HC structure has a much greater comparative advantage over the conglomerate structure, especially when the correlation between the returns to the assets of the merging firms is highly positive. This is because, the pooling of asset risks by a conglomerate is most expected for highly negative correlations, when the HC is unable to muster a significantly higher value due to equity risk pooling. As the correlation of merging firms increases, the equity risk pooling becomes much more expected, resulting ultimately in a 94 percent improvement over the conglomerate for $\rho = .8$. This result seems to be consistent with the anecdotal evidence, which suggests that at the time of a merger, the firms involved usually remain subsidiaries, rather than form a single entity with one tier of debt and equity claims.

5.2 Converting a Firm into a One-sub Holding Company

In this subsection, we examine the improvement in the NPV of converting an independent firm ($e = 1$) into a one-sub HC, when the costs of formation are zero. We consider the same tax rate, $\tau = .3$, interest rate $r = .1$, and firm parameters, $\sigma_1^2 = .25$ and $\mu_1 = .1$. But, we use 15 different normal legal fees, $C_1^c = cF_1$, where c is a positive fraction with values .01, .02, ..., .15. We present these results in Table 4. Columns 2 and 6 of Table 4 show that the one-sub HC optimally transfers debt from the subsidiary to the parent entity. The shifting of the debt burden from the first tier to the second tier increases as the normal legal fee increases. The increase in debt capacity, which is given by the difference between Column 4 and Column 6 and which represents the amount of debt shifted from the first to the second tier, increases with the legal fee parameter c. Column 8 shows a positive debt capacity of the parent HC, even when the subsidiary's debt is restricted to be equal to the independent firm's optimal debt (Column 6). This restricted

Table 4
Numerically Estimated Values For One-sub HC

	HC				Firm		HC*		Improvement		Percent of (5)-(7)	
c	D_1^{A1} (in \$)	D_p^{A1} (in \$)	2+3 (in \$)	v_p^{A1} (in \$)	D_1^{A2} (in \$)	v_1^{A2} (in \$)	D_p^* (in \$)	v_p^{A1*} (in \$)	$\frac{(5)-(7)}{Abs(7)}$ (in %)	$\frac{(9)-(7)}{Abs(7)}$ (in %)	Bank. Cost Saving	Two Tiers
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
.01	.94	.16	1.10	.11	1.07	.09	.03	.10	15	8	43	57
.02	.88	.22	1.10	.11	1.07	.09	.03	.10	24	8	82	18
.03	.87	.19	1.06	.08	1.02	.06	.05	.07	46	21	69	31
.04	.88	.21	1.09	.11	1.01	.08	.09	.10	39	24	37	63
.05	.78	.24	1.02	.06	.94	.02	.08	.04	143	83	48	52
.06	.78	.25	1.03	.09	.95	.05	.09	.07	80	45	59	41
.07	.78	.23	1.00	.07	.90	.03	.13	.06	154	112	35	65
.08	.78	.24	1.02	.08	.91	.04	.12	.07	105	66	35	65
.09	.66	.30	.96	.03	.83	-.01	.15	.02	315	246	15	85
.10	.66	.29	.95	.07	.83	.02	.15	.06	183	143	19	81
.11	.66	.27	.94	.03	.83	-.01	.13	.02	467	311	36	64
.12	.66	.25	.91	.02	.75	-.03	.18	.01	169	154	-7	107
.13	.66	.24	.90	.03	.75	-.02	.19	.02	262	231	-10	110
.14	.66	.26	.91	.02	.75	-.02	.17	.02	205	177	-11	111
.15	.65	.25	.91	.04	.74	-.01	.17	.03	783	633	-4	104

parent HC debt capacity increases with the bankruptcy cost parameter, c. Observe that the parent HC creates a greater debt capacity (at competitive costs) than the improvement in the NPV (which accounts for all costs) of the HC, as compared to that of the independent firm. Columns 10 and 11 show that the improvement in the value of the HC, when its subsidiary debt is unrestricted, is higher than the value otherwise. Like in the two-sub HC, we decompose the total value created by the formation of an HC into two components: (i) due to the bankruptcy cost savings (Column 12) and (ii) due to two tiers (Column 13). The value created by a two-tier structure, even if the expected bankruptcy cost savings are excluded, appears quite substantial.

The main point here is that an independent firm could form itself into an HC by splitting its debt across two tiers. This involves an increase in the *ex ante* formation cost, but a decrease in the *ex post* bankruptcy cost. Splitting the debt into two entities permits selective restructuring, for example, when the parent entity (not the subsidiary) defaults. In this case, the cost of restructuring the entire firm can be circumvented. The lower DOW value and hence the lower bankruptcy cost can thus result in a higher value of the one-sub HC than the independent firm. But, there is a tradeoff between this extra value of forming a one-sub HC and the extra cost of forming two entities, as compared to that for the independent firm as one entity.

6 Conclusion

We introduced a novel idea: Debtholders of a firm have a put option to walk out (DOW) of a bankruptcy process, when the value of assets of the firm in the state of default is less than the transaction cost needed to recover any value from the assets.

The value of DOW plays a crucial role in our model in choosing whether independently operating firms should be merged into a holding company (HC) or a conglomerate. Merging into a conglomerate can generate a smaller DOW value than the total of DOW values of the independent firms. A merger, however, can create value from diversification of risk of assets possible in a conglomerate. It is not optimal to merge independent firms with high DOW values into a conglomerate. The DOW value is high when the debtholders' out-of-pocket expense during bankruptcy is high. The incremental value from diversification of risk of assets of such independent firms does not compensate the drop in the DOW value due to their conglomeration.

The HC structure the most flexible organization. With no parent debt, the HC mimics a set of independent subsidiaries, and with no subsidiary debt, it is simply a conglomeration (merger) of independent firms. A multi-tier holding company capital structure (as opposed to conglomerate) may be optimal for technological and industrial assets which impute large bankruptcy restructuring costs. Investing in such assets within separate subsidiaries of a holding company can generate higher values as selective exercise of DOWs is possible and valuable. We determined the optimal capital structure of a holding company for its each subsidiary and parent.

The Appendix

Proof of Lemma 1: The after-tax payoff of an independent firm or the conglomerate, e, is $X_e - \tau(X_e - F_e)^+$. If $X_e > F_e$, then $X_e - F_e - \tau(X_e - F_e)^+ = (1-\tau)(X_e - F_e) > 0$. Suppose that $X_e - F_e - \tau(X_e - F_e)^+ > 0$, but $X_e \leq F_e$. Then, we have a contradiction, which proves the first inequality.

The second inequality in (9) follows for $X_c \leq F_h$. For $X_c > F_h$, (8) shows that $X_p - F_p > 0$ and $X_p - \tau\left(X_c - F_h\right)^+ - F_p = (1-\tau)(X_c - F_h) > 0$. ∎

Proof of Proposition 1: The asset payoff X_e of firm e is lognormally distributed under Q:

$$X_e = X_e(0)Exp\left\{r - \frac{1}{2}\sigma_e^2 + \sigma_e Z_e(1)\right\}, \tag{A1}$$

where the distribution of $Z_e(1) = \int_0^1 dZ_e(t)$ is unit normal. Then, the market value of firm e, denoted by V_e, is given by

$$\begin{aligned}
V_e &= E\left(\beta\left[(1-\tau)(X_e - F_e)^+ + \min(X_e, F_e) + (fF_e - X_e)^+ - fF_eJ_e\right]\right), \\
&= E\left(\beta\left[-\tau(X_e - F_e)^+ + X_e - fF_e + (fF_e - X_e)^+ + fF_e(1 - J_e)\right]\right), \\
&= E\left(\beta\left[-\tau(X_e - F_e)^+ + (X_e - fF_e)^+ + fF_e(1 - J_e)\right]\right), \\
&= -\tau Y_e(F_e) + Y_e(fF_e) + \beta fF_e \Pr(X_e > F_e), \qquad \text{(A2)}
\end{aligned}$$

where $Y_e(F)$ is the value of a European call option written on the assets of entity e with an exercise price $F = F_e,\ fF_e$. Using the standard techniques (see, e.g., Ingersoll (1987)), this call option value can be written as

$$Y_e\left(F\right) = \left[X_e(0)N\left(d_1\left(F\right)\right) - \beta F N\left(d_2(F)\right)\right], \tag{A3}$$

where

$$d_1\left(F\right) = \frac{1}{\sigma_e}\left[ln\left(\frac{X_e(0)}{\beta F}\right) + \frac{1}{2}\sigma_e^2\right],\ \ d_2(F) = d_1(F) - \sigma_e,\ F = F_e, fF_e. \tag{A4}$$

Thus, V_e can be written as a function of F_e:

$$V_e\left(F_e\right) = -\tau\left[X_e(0)N\left(d_1\left(F_e\right)\right) - \beta F_e N\left(d_2(F_e)\right)\right]$$

$$+\left[X_e(0)N\left(d_1\left(fF_e\right)\right)-\beta fF_eN\left(d_2(fF_e)\right)\right]+\beta fF_eN\left(d_2(F_e)\right). \tag{A5}$$

We need to show that there exists a unique solution, $F_e > 0$, which maximizes $V_e\left(F_e\right)$. Denote the standard normal density by

$$n(z) \equiv \frac{1}{\sqrt{2\pi}}\text{Exp}\left(-\frac{1}{2}z^2\right). \tag{A6}$$

It follows that

$$\frac{\partial d_1\left(z\right)}{\partial z}=\frac{\partial d_2\left(z\right)}{\partial z}=-\frac{1}{z\sigma_e},\quad \frac{\partial n\left(z\right)}{\partial z}=-n(z)z. \tag{A7}$$

Then the first order condition of maximization of $V_e\left(F_e\right)$ with respect to F_e is

$$\begin{aligned} 0 &= \frac{\partial V_e\left(F_e\right)}{\partial F_e} \\ &= \beta\left[\tau N\left(d_2(F_e)\right)-fN\left(d_2(fF_e)\right)\right]+\beta fN(d_2(F_e))-\frac{\beta f}{\sigma_e}n(d_2(F_e)) \\ \Longleftrightarrow\ 0 &= (\tau+f)-f\frac{N(d_2(fF_e))}{N(d_2(F_e))}-\frac{f}{\sigma_e}\frac{n(d_2(F_e))}{N(d_2(F_e))}\equiv G(F_e), \end{aligned} \tag{A8}$$

where we have used the derivative of a European call option value with respect to the exercise price (see, e.g., Ingersoll (1987)),

$$\frac{\partial Y(F)}{\partial F}=-\beta N(d_2(F)). \tag{A9}$$

We first show that the last term in (A8) is monotonically decreasing in F_e, i.e., $n(d_2(z))/N(d_2(z))$ is increasing in z:

$$\begin{aligned} \frac{d}{dz}\left(\frac{n(d_2(z))}{N(d_2(z))}\right) &= \left(\frac{n(d_2(z)(-d_2(z))N(d_2(z))-n(d_2(z))^2}{N(d_2(z))^2}\right)\frac{d\left[d_2(z)\right]}{dz} \\ &= \frac{n(d_2(z)}{N(d_2(z))}\left[d_2(z)+\frac{n(d_2(z))}{N(d_2(z))}\right]\frac{1}{\sigma_e z}>0, \end{aligned} \tag{A10}$$

for all $z > 0$ since $E(X|X < x) = -\frac{n(x)}{N(x)} < x$, i.e., $x + \frac{n(x)}{N(x)} > 0$ for a standard normal variate, X for all x, $-\infty < x < \infty$. To show that the second term in (A8) is monotonically decreasing in F_e, observe that

$$\begin{aligned} \frac{d}{dz}\left(\frac{N(d_2(fz))}{N(d_2(z))}\right) &= \frac{n(d_2(fz))\left(-\frac{f}{fz\sigma_e}\right)N(d_2(z))-n(d_2(z))\left(-\frac{1}{z\sigma_e}\right)N(d_2(fz))}{N(d_2(z))^2}>0 \\ \Longleftrightarrow &\quad \frac{n(d_2(fz)}{N(d_2(fz))}<\frac{n(d_2(z))}{N(d_2(z))}, \end{aligned} \tag{A11}$$

which is true by (A10) since $fz < z$. Thus, $G(F_e)$ is a monotonically decreasing function and

$$\lim_{F_e \to 0} G(F_e) = \tau > 0, \quad \lim_{F_e \to \infty} G(F_e) = -\infty, \tag{A12}$$

which shows that there exists a unique $F_e > 0$ which solves $G(F_e) = 0$ as well as $N(d_2(F_e))G(F_e) = 0$ by (A8). Further (using $G(F_e) = 0$),

$$\begin{aligned} \frac{d[N(d_2(F_e))G(F_e)]}{dF_e} &= -G(F_e)\frac{n(d_2(F_e))}{\sigma_e F_e} + N(d_2(F_e))\frac{dG(F_e)}{dF_e} \\ &= N(d_2(F_e))\frac{dG(F_e)}{dF_e} < 0, \end{aligned} \tag{A13}$$

which means that the solution to (A8) is also the maximizer of $V_e(F_e)$. ∎

Proof of Lemma 2: Observe that

$$\begin{aligned} W &= E\left\{\beta\left(S_p(X_p, F_p) - \sum_{e=1}^{m} I_e\alpha_e - I_p\alpha_p + B_p(X_p, F_p) + \sum_{e=1}^{m} B_e(X_e, B_e)\right)\right\} \\ &= E\left\{\beta\left(X_p - \tau\,(X_c - F_h)^+ + \sum_{e=1}^{m} \text{min}(X_e, F_e)\right) - \sum_{e=1}^{m} C_e - C_p\right\} \\ &= E\left\{\beta\left(X_p - \tau\,[X_c - \text{min}\,(X_c, F_h)] + \sum_{e=1}^{m}\left[X_e - (X_e - F_e)^+\right]\right) - \sum_{e=1}^{m} C_e - C_p\right\} \\ &= E\left\{\beta\,((1-\tau)X_c + \tau\text{min}\,(X_c, F_h)) - \sum_{e=1}^{m} C_e - C_p\right\}, \end{aligned}$$

where we have used the definition of X_p from (2). ∎

Proof of Proposition 2: By the supposition there exists $\alpha_e \geq 0$, satisfying (14), such that $\hat{F}_e$ is the optimal debt of independent firm $e = 1, 2, ..., m$. Let the HC fix its subsidiary debts equal to $\hat{F} \equiv (\hat{F}_1, ..., \hat{F}_m)$, and let $X_p(\hat{F}) = \sum_{e=1}^{m}(X_e - \hat{F}_e)^+$. Given $\hat{F}$, maximizing the HC's objective in (17) is equivalent to solving,

$$\underset{F_p \geq 0}{\text{Maximize}} \quad \tau E\left\{\beta\text{min}\left(X_c, F_p + \hat{F}_s\right)\right\} - E[\beta\text{min}(X_p(\hat{F}), fF_p)J_p], \tag{A14}$$

where $\hat{F}_s = \sum_{e=1}^{m} \hat{F}_e$. We first prove the following lemma.

Lemma 3: The optimal solution to (A13), denoted by F_p^*, is positive.

Proof of Lemma 3: Let $g_c(X_c)$ and $g_p(X_p(\hat{F}))$ be the probability density function of X_c and $X_p(\hat{F})$, respectively, induced by $(X_1, X_2, ..., X_m)$. Let the objective function in (A13) be denoted by $H(F_p)$. Then, it is sufficient to show that

$$\left.\frac{dH(F_p)}{dF_p}\right|_{F_p=0} > 0, \tag{A15}$$

i.e., a nonzero level of parent debt increases the value of the objective function. Now

$$
\begin{aligned}
H(F_p) &= \beta\tau\int_{\hat{F}_s}^{\hat{F}_s+F_p} X_c g_c(X_c)dX_c + \beta\tau\int_{\hat{F}_s+F_p}^{\infty}(\hat{F}_s+F_s)g_c(X_c)dX_c \\
&\quad -\beta\int_0^{fF_p} X_p g_p(X_p)dX_p - \beta\int_{fF_p}^{F_p} fF_p g_p(X_p)dX_p. \qquad \text{(A16)}
\end{aligned}
$$

Differentiating $H(F_p)$ with respect to F_p [18]

$$
\begin{aligned}
\frac{dH(F_p)}{dF_p} &= \beta\tau(\hat{F}_s+F_p)g_c(\hat{F}_s+F_p) + \beta\tau\int_{\hat{F}_s+F_p}^{\infty} g_c(X_c)dX_c - \beta\tau(\hat{F}_s+F_p)g_c(\hat{F}_s+F_p) \\
&\quad -\beta fF_p g_p(fF_p) - \beta\int_{fF_p}^{F_p} f g_p(X_p)dX_p - \beta fF_p g_p(F_p) + \beta fF_p g_p(fF_p). \qquad \text{(A17)}
\end{aligned}
$$

Setting $F_p = 0$ in (A16),

$$
\left.\frac{dH(F_p)}{dF_p}\right|_{F_p=0} = \beta\tau\int_{\hat{F}_s}^{\infty} g_c(X_c)dX_c > 0. \qquad \text{(A18)}
$$

■

Proof of Proposition 2 Continued: Since, by Lemma 3, $F_p^* > 0$,

$$
\tau E\left\{\beta \min\left(X_c, F_p^* + \hat{F}_s\right)\right\} - E[\beta\min(X_p(\hat{F}), fF_p^*)J_p] > \tau E\left\{\beta\min\left(X_c, \hat{F}_s\right)\right\}. \qquad \text{(A19)}
$$

As discussed earlier $F_p^* > 0$ if and only if $D_p > 0$. We have to show, however, that there exists $\alpha_p > 0$ such that the HC chooses F_p^* as its optimal parent debt, given $\hat{F}$, i.e., show that $W(F_p^*, \hat{F}) > W(0, \hat{F})$. Observe that for $\alpha_p = 0$ [but, $\alpha_e > 0,\ (e = 1, 2..., m)$ and satisfying (14)],

$$
\begin{aligned}
W(F_p^*, \hat{F}) &= \overline{W} + \tau E\left[\beta\min\left(X_c, \hat{F}_s + F_p^*\right)\right] - E\left[\sum_{e=1}^{m} C_e(X_e, \hat{F}_e) + \beta\min(X_p(\hat{F}), fF_p^*)J_p\right], \\
&> \overline{W} + \tau E\left[\beta\min\left(X_c, \hat{F}_s\right)\right] - E\left[\beta\sum_{e=1}^{m} C_e(X_e, \hat{F}_e)\right], \\
&= W(0, \hat{F}),
\end{aligned}
$$

where the first inequality follows from (A18). Thus, by continuity, there exists $\alpha_p > 0$ such that $W(F_p^*, \hat{F}) > W(0, \hat{F})$. In particular, α_p is less than the difference between the left and the right sides of (A18). ■

[18] The Leibnitz's rule for differentiating an integral is:

$$
\frac{d}{dx}\int_{a_1(x)}^{a_2(x)} F(u,x)du = \int_{a_1(x)}^{a_2(x)} \frac{dF(u,x)}{dx}du + F(a_2(x),x)\frac{da_2(x)}{dx} - F(a_1(x),x)\frac{da_1(x)}{dx}.
$$

Proof of Proposition 3: Let $\hat{F}_e$ be the optimal debt of independent firm, e, and $(\tilde{F}_p, \tilde{F} \equiv (\tilde{F}_1, \tilde{F}_2, ..., \tilde{F}_m))$ be the optimal debt levels of the HC. We first show that the market value of tax benefits of the HC is strictly greater than the sum of the market values of tax benefits of the independent firms, if the HC sets subsidiary e's debt equal to $\hat{F}_e$, $e = 1, 2, ..., m$. Defining $\hat{F}_s \equiv \hat{F}_1 + \hat{F}_2 + ... + \hat{F}_m$,

$$\begin{aligned} X_p &= \sum_{e=1}^{m} (X_e - \hat{F}_e)^+ \\ &= \sum_{e=1}^{m} \left(X_e - \min(X_e, \hat{F}_e) \right) \\ &= X_c - \sum_{e=1}^{m} \min(X_e, \hat{F}_e) \\ &= \min(X_c, \hat{F}_s + F_p) + (X_c - \hat{F}_s - F_p)^+ - \sum_{e=1}^{m} \min(X_e, \hat{F}_e), \end{aligned}$$

which implies that [using $(X_p - F_p)^+ \geq (X_c - \hat{F}_s - F_p)^+$ from (7)]

$$\begin{aligned} \min(X_c, \hat{F}_s + F_p) &= X_p - (X_c - \hat{F}_s - F_p)^+ + \sum_{e=1}^{m} \min(X_e, \hat{F}_e) \\ &= \min(X_p, F_p) + (X_p - F_p)^+ - (X_c - \hat{F}_s - F_p)^+ + \sum_{e=1}^{m} \min(X_e, \hat{F}_e) \\ &\geq \min(X_p, F_p) + \sum_{e=1}^{m} \min(X_e, \hat{F}_e), \\ &\geq \sum_{e=1}^{m} \min(X_e, \hat{F}_e), \end{aligned} \tag{A20}$$

where the inequalities are strict for some *ex post* realizations of X_e and for some $F_e > 0$, $e = 1, 2, ..., m, p$, which imply that

$$E\left[\beta \min(X_c, \hat{F}_s + F_p)\right] > E\left[\beta \min(X_c, \hat{F}_s)\right] > E\left[\beta \sum_{e=1}^{m} \min(X_e, \hat{F}_e)\right]. \tag{A21}$$

Since $(\tilde{F}_p, \tilde{F})$ are the optimal debts of the HC

$$\begin{aligned} \mathcal{V}(A1) &= W(\tilde{F}_p, \tilde{F}) \\ &\geq W(F_p^*(\hat{F}), \hat{F}) \\ &> W\left(0, \hat{F}\right) \\ &= \overline{W} + \tau E[\beta \min(X_c, \hat{F}_1 + \hat{F}_2 + ... + \hat{F}_m)] - \sum_{e=1}^{m} E[C_e(X_e, \hat{F}_e)] \\ &> \overline{W} + \tau E\left[\beta \sum_{e=1}^{m} \min(X_e, \hat{F}_e)\right] - \sum_{e=1}^{m} E[C_e(X_e, \hat{F}_e)] \\ &= \sum_{e=1}^{m} V_e\left(\hat{F}_e\right) = \mathcal{V}(A2), \end{aligned} \tag{A22}$$

where the first strict inequality follows from Proposition 2 and the second from (A20). The last equality in (A21) obtains because maximizing $V_e(F_e)$ with respect to F_e, independently for $e = 1, 2, ..., m$, yields the same solution $\hat{F}_e$ as maximizing $\sum_{e=1}^{m} V_e(F_e)$ with respect to $(F_1, F_2, ..., F_m)$. ∎

Proof of Proposition 4: If the HC pays taxes based on entity income, then [using (8)] in state $\{X_p > F_p\} = \{X_c > \sum_{e=1}^{m} \min(X_e, F_e) + F_p\}$, the HC's taxes equal $\tau(X_p - F_p)$ if it is levered with face values $(F_1, F_2, ..., F_m, F_p)$, as compared to τX_c if it is not levered; resulting in tax benefits of leverage, $\tau X_c - \tau(X_p - F_p) = \tau(X_c - X_p + F_p) = \tau\left[\sum_{e=1}^{m} \min(X_e, F_e) + F_p\right]$. In the complementary state $\{X_c \leq \sum_{e=1}^{m}(X_e, F_e) + F_p\}$, the HC's income taxes equal 0 if it is levered, as compared to τX_c if it is unlevered; resulting in tax benefits of leverage, τX_c. Thus, the HC's tax benefits based on entity incomes is $\tau \min\left(X_c,\ \sum_{e=1}^{m} \min(X_e, F_e) + F_p\right)$. Further,

$$X_c - \sum_{e=1}^{m} \min(X_e, F_e) = \sum_{e=1}^{m} [X_e - \min(X_e, F_e)] = \sum_{e=1}^{m} (X_e - F_e)^+ \geq 0, \tag{A23}$$

which implies that the HC's tax benefits based on entity incomes is at least equal to the independent firms' tax benefits:

$$\begin{aligned} \tau \min\left(X_c,\ \sum_{e=1}^{m} \min(X_e, F_e) + F_p\right) &\geq \tau \min\left(X_c,\ \sum_{e=1}^{m} \min(X_e, F_e)\right) \\ &= \tau\left(\sum_{e=1}^{m} \min(X_e, F_e)\right), \end{aligned} \tag{A24}$$

where the inequality is strict for some *ex post* realizations. Then if $\hat{F}$ is the vector of optimal debts chosen by the independent firms and if the HC chooses its subsidiary debts equal to $\hat{F}$,

$$\begin{aligned} W\left(0, \hat{F}\right) &= \overline{W} + \tau E\left[\beta \min\left(X_c,\ \sum_{e=1}^{m} \min(X_e, \hat{F}_e)\right)\right] - \sum_{e=1}^{m} E[C_e(X_e, \hat{F}_e)] \\ &= \overline{W} + \tau E\left[\beta \sum_{e=1}^{m} \min(X_e, \hat{F}_e)\right] - \sum_{e=1}^{m} E[C_e(X_e, \hat{F}_e)], \end{aligned}$$

and hence the result follows from (A21). ∎

Proof of Proposition 5: (a) Since the tax benefits of the alternatives are equal for all $F_s \leq F_c$, we just need to show that the expected bankruptcy cost of the HC is smaller than that of the conglomerate for some value of F_s with $0 < F_s < F_c$. The HC's expected bankruptcy cost, $\delta(F_s)$, expressed as a function of its subsidiary debt is given by:

$$\begin{aligned} \delta(F_s) &\equiv \delta_s(F_s) + \delta_p(F_s) \\ &\equiv \beta f F_s \Pr(X_1 < F_s) - \beta E(fF_s - X_1)^+ \\ &\quad + \beta f F_p \Pr(X_p < F_p) - \beta E(fF_p - X_p)^+, \end{aligned} \tag{A25}$$

where $F_p = F_c - F_s$ and $X_p = (X_1 - F_s)^+$. Observe that

$$\lim_{F_s \to 0} \delta(F_s) = \lim_{F_s \to F_c} \delta(F_s) = \beta f F_c \Pr(X_1 < F_c) - (fF_c - X_1)^+, \tag{A26}$$

i.e, the bankruptcy cost of the HC with no subsidiary or no parent is equal to that of the conglomerate. To prove that <u>nonzero</u> levels of debt of both the parent <u>and</u> the subsidiary decrease the HC's bankruptcy cost, we show that

$$\left.\frac{d\delta(F_s)}{dF_s}\right|_{F_s=0} < 0, \text{ and } \left.\frac{d\delta(F_s)}{dF_s}\right|_{F_s=F_c} > 0. \tag{A27}$$

We now use the put-call parity for evaluating the DOWs in (A24):

$$\begin{aligned}\delta_s(F_s) &= \beta f F_s[1-N(d_2(F_s))] - \underbrace{[\beta f F_s - X_1(0) + \underbrace{X_1(0)N(d_1(fF_s)) - \beta f F_s N(d_2(fF_s))}_{\text{Call Option Value } (Y(fF_s))}]}_{\text{The DOW Value}} \\ &= -\beta f F_s N(d_2(F_s)) + X_1(0) - [X_1(0)N(d_1(fF_s)) - \beta f F_s N(d_2(fF_s))] \\ &= f\underbrace{[X_1(0)N(d_1(F_s)) - \beta F_s N(d_2(F_s))]}_{Y(F_s)} + X_1(0)(1 - fN(d_1(F_s))) \\ &\quad - \underbrace{[X_1(0)N(d_1(fF_s)) - \beta f F_s N(d_2(fF_s))]}_{Y(fF_s)},\end{aligned} \tag{A28}$$

where $d_1(\cdot)$ and $d_2(\cdot)$ are defined in (A4), $\Pr(X_1 < z) = 1 - N(d_2(z))$ and $Y(\cdot)$ in (A3). We now evaluate δ_p [letting $g(X_1)$ denote the density function of X_1]:

$$\begin{aligned}\delta_p &= \beta \int_{\{X_p<F_p\}} \min(X_p, fF_p)g(X_1)dX_1 \\ &= \beta \int_{\{X_1<F_s;X_p<F_p\}} \min(X_p, fF_p)g(X_1)dX_1 + \beta \int_{\{X_1\geq F_s;X_p<F_p\}} \min(X_p, fF_p)g(X_1)dX_1 \\ &= \beta \int_{\{X_1\geq F_s;X_p<F_p\}} \min(X_p, fF_p)g(X_1)dX_1 \\ &= \beta \int_{\{F_c>X_1\geq F_s\}} \min(X_1 - F_s, fF_p)g(X_1)dX_1 \\ &= \beta \int_{\{F_c>X_1\geq F_s\}} \min(X_1, F_s + fF_p)g(X_1)dX_1 - \beta \int_{\{F_c>X_1\geq F_s\}} F_s g(X_1)dX_1 \\ &= \beta \int_{F_s}^{F_s+fF_p} X_1 g(X_1)dX_1 + \beta \int_{F_s+fF_p}^{F_c} (F_s + fF_p)g(X_1)dX_1 - \beta \int_{F_s}^{F_c} F_s g(X_1)dX_1 \\ &= \beta \int_{F_s}^{F_s+fF_p} X_1 g(X_1)dX_1 + \beta f F_p \int_{F_s+fF_p}^{F_c} g(X_1)dX_1 - \beta F_s \int_{F_s}^{F_s+fF_p} g(X_1)dX_1 \\ &= X_1(0)N(d_1(F_s)) - X_1(0)N(d_1(F_s + fF_p)) + \beta f F_p[N(d_2(F_s + fF_p)) - N(d_2(F_c))] \\ &\quad -\beta F_s[N(d_2(F_s)) - N(d_2(F_s + fF_p))]\end{aligned}$$

$$= \underbrace{X_1(0)N(d_1(F_s)) - \beta F_s N(d_2(F_s))}_{Y(F_s)} - \underbrace{[X_1(0)N(d_1(F_s+fF_p)) - \beta(F_s+fF_p)N(d_2(F_s+fF_p))]}_{Y(F_s+fF_p)} - \beta f F_p N(d_2(F_c)), \tag{A29}$$

where $\beta\int_z^\infty X_1 g(X_1)dX_1 = X_1(0)N(d_1(z))$ and $\int_z^\infty g(X_1)dX_1 = N(d_2(z))$. Differentiating δ_s in (A27) with respect to F_s [using (A9)],

$$\frac{d\delta_s}{dF_s} = -\beta f N(d_2(F_s)) + \beta f N(d_2(fF_s)) + \frac{fX_1(0)}{\sigma_1 F_s} n(d_1(F_s)), \tag{A30}$$

and δ_p in (A28) with respect to F_s [using $F_p = F_c - F_s$],

$$\frac{d\delta_p}{dF_s} = -\beta N(d_2(F_s)) + \beta(1-f)N(d_2(F_s+fF_p)) + \beta f N(d_2(F_c)). \tag{A31}$$

To show (A26), we first apply L'Hopital's rule to evaluate

$$\begin{aligned} \lim_{F_s \to 0} \frac{n(d_2(F_s))}{F_s} &= \lim_{F_s \to 0} \frac{d[n(d_2(F_s))]}{dF_s} \\ &= \lim_{F_s \to 0} \frac{n(d_2(F_s))d_2(F_s)}{F_s\sigma_1} \\ &= \left(\lim_{F_s \to 0} \frac{n(d_2(F_s))}{F_s}\right)\left(\lim_{F_s \to 0} \frac{d_2(F_s)}{\sigma_1}\right), \end{aligned} \tag{A32}$$

i.e.,

$$\left(\lim_{F_s \to 0} \frac{n(d_2(F_s))}{F_s}\right)\left(\lim_{F_s \to 0} \frac{d_2(F_s)}{\sigma_1} - 1\right) = 0, \tag{A33}$$

which implies that

$$\lim_{F_s \to 0} \frac{n(d_2(F_s))}{F_s} = 0, \tag{A34}$$

since

$$\lim_{F_s \to 0} \frac{d_2(F_s)}{\sigma_1} \neq 1. \tag{A35}$$

Using (A29) and (A30), $d\delta/dF_s = d\delta_s/dF_s + d\delta_p/dF_p$ and

$$\begin{aligned} \lim_{F_s \to 0} \frac{d\delta}{dF_s} &= -\beta f + \beta f + 0 - \beta + \beta(1-f)N(d_2(fF_c)) + \beta f N(d_2(F_c)) \\ &= \beta[N(d_2(fF_c)) - 1] + \beta f[N(d_2(F_c) - N(d_2(fF_c))] \\ &< 0. \end{aligned} \tag{A36}$$

and

$$\begin{aligned} \lim_{F_s \to F_c} \frac{d\delta}{dF_s} &= -\beta f N(d_2(F_c)) + \beta f N(d_2(fF_c)) + \frac{fX_1(0)}{\sigma_1 F_c} n(d_1(F_c)) \\ &\quad -\beta N(d_2(F_c)) + \beta(1-f)N(d_2(F_c)) + \beta f N(d_2(F_c)) \\ &> 0. \end{aligned} \tag{A37}$$

Thus, the minimized HC bankruptcy cost is strictly less than that of the conglomerate by (A25), i.e.,

$$\mathcal{V}(A1|F_s + F_p = F_c) > \mathcal{V}(A3|F_c), \text{ given } \alpha_1 = \alpha_c = \alpha_p = 0. \tag{A38}$$

(b) Due to the strict inequality in (A37), there exist $\alpha_1 > 0,\ \alpha_c > 0$ and $\alpha_p > 0$ such that (A37) holds. ∎

Proposition 6: The bankruptcy cost of the HC without the DOW is given by

$$\begin{aligned}\delta(F_s) &= \beta f F_s \Pr(X_1 < F_s) + \beta f F_p \Pr(X_p < F_p)\\ &= \beta f F_s \Pr(X_1 < F_s) + \beta f F_p \Pr(X_p < F_p|X_1 < F_s) \Pr(X_1 < F_s)\\ &\quad + \beta f F_p \Pr(X_p < F_p|X_1 \geq F_s) \Pr(X_1 \geq F_s)\\ &= \beta f F_s \Pr(X_1 < F_s) + \beta f F_p \Pr(X_1 < F_s) + \beta f F_p \Pr(F_s \leq X_1 < F_c)\\ &= \beta f F_s[1 - N(d_2(F_s))] + \beta f F_p[1 - N(d_2(F_c))].\end{aligned} \tag{A39}$$

Differentiating $\delta(F_s)$,

$$\frac{d\delta(F_s)}{dF_s} = \frac{\beta f}{\sigma_1}\frac{n(d_2(F_s))}{F_s} - \beta f[1 - N(d_2(F_c))]. \tag{A40}$$

(a) Observe that, given $\delta(0) = \delta(F_c)$, some subsidiary debt is optimal (when $\alpha_e = 0$), i.e.,

$$\lim_{F_s \to 0} \frac{d\delta(F_s)}{dF_s} = -\beta f[1 - N(d_2(F_c))] < 0. \tag{A41}$$

While for all reasonable parameters, our numerical estimates show that

$$\lim_{F_s \to F_c} \frac{d\delta(F_s)}{dF_s} = \frac{\beta f}{\sigma_1}\frac{n(d_2(F_c))}{F_c} - \beta f[1 - N(d_2(F_c))] > 0, \tag{A42}$$

we do not need this for proving that $\mathcal{V}(A1|F_p = F_c - F_s) > \mathcal{V}(A3)$. (b) Further, there exists, $\alpha_e > 0$ such that $\mathcal{V}(A1|F_p = F_c - F_s) > \mathcal{V}(A3)$. ∎

Proposition 7: The HC's expected bankruptcy cost, $\delta(F_s)$, expressed as a function of its subsidiary debt is given by:

$$\delta(F_s) = \underbrace{\beta f_s \Pr(X_1 < F_s) - \beta E(f_s - X_1)^+}_{\delta_s(F_s)} + \underbrace{\beta f_p \Pr(X_p < F_p) - \beta E(f_p - X_p)^+}_{\delta_p(F_s)}, \tag{A43}$$

where $F_p = F_c - F_s$ and $X_p = (X_1 - F_s)^+$. Observe that

$$\begin{aligned}\lim_{F_s \to 0} \delta(F_s) &= \beta f_p \Pr(X_1 < F_c) - \beta E(f_p - X_1)^+ = \beta f_c \Pr(X_1 < F_c) - \beta E(f_c - X_1)^+\\ &= \beta f_s \Pr(X_1 < F_c) - \beta E(f_s - X_1)^+ = \lim_{F_s \to F_c} \delta(F_s).\end{aligned} \tag{A44}$$

Replacing fF_e with f_e in (A27) and (A28):

$$\delta(F_s) = \delta_s(F_s) + \delta_p(F_p)$$

$$= \beta f_s[1 - N(d_2(F_s))] - [\beta f_s - X_1(0) + X_1(0)N(d_1(f_s)) - \beta f_s N(d_2(f_s))]$$

$$+ \quad X_1(0)N(d_1(F_s)) - \beta F_s N(d_2(F_s)) -$$

$$[X_1(0)N(d_1(F_s + f_p)) - \beta(F_s + f_p)N(d_2(F_s + f_p))] - \beta f_p N(d_2(F_c)). \tag{A45}$$

Differentiating $\delta(F_s)$ with respect to F_s,

$$\frac{d\delta}{dF_s} = \frac{\beta f_s n(d_2(F_s))}{\sigma_1 F_s} - \beta N(d_2(F_s)) + \beta N(d_2(F_s + f_p)), \tag{A46}$$

(a) Clearly [using (A33)],

$$\lim_{F_s \to 0} \frac{d\delta}{dF_s} = 0 - \beta + \beta N(d_2(f_p)) < 0. \tag{A47}$$

Further, although we do not need,

$$\lim_{F_s \to F_c} \frac{d\delta}{dF_s} = \frac{\beta f_s n(d_2(F_c))}{\sigma_1 F_c} - \beta N(d_2(F_c)) + \beta N(d_2(F_c + f_p)) > 0, \tag{A48}$$

for some $f_s = f_c = f_p > 0$. (b) There exits $\alpha_e > 0$ such that $\mathcal{V}(A1|F_p = F_c - F_s) > \mathcal{V}(A3)$. ∎

Proposition 8: The HC's expected bankruptcy cost, $\delta(F_s)$, expressed as a function of its subsidiary debt is given by:

$$\delta(F_s) = \underbrace{\beta f_s \Pr(X_1 < F_s)}_{\delta_s(F_s)} + \underbrace{\beta f_p \Pr(X_p < F_p)}_{\delta_p(F_s)}, \tag{A49}$$

where $F_p = F_c - F_s$ and $X_p = (X_1 - F_s)^+$. Then, clearly,

$$\lim_{F_s \to 0} \delta(F_s) \quad = \quad \beta f_c \Pr(X_1 < F_c) = \lim_{F_s \to F_c} \delta(F_s). \tag{A50}$$

Replacing fF_e with f_e in (A38):

$$\begin{aligned} \delta(F_s) &= \delta_s(F_s) + \delta_p(F_p) \\ &= \beta f_s[1 - N(d_2(F_s))] + \beta f_p[1 - N(d_2(F_c))]. \end{aligned} \tag{A51}$$

(a) Differentiating $\delta(F_s)$ with respect to F_s,

$$\frac{d\delta(F_s)}{dF_s} = \frac{\beta f_s}{\sigma_1} \frac{n(d_2(F_s))}{F_s} \geq 0, \; \forall \, F_s, \tag{A52}$$

which means that holding any subsidiary debt is suboptimal. (b) This is obvious. ∎

Proof of Proposition 9: (a) Let, for every level of debt, F_c, chosen by the conglomerate, the independent firms' choices of debt, F_e, satisfy $F_c = \sum_{e=1}^{m} F_e$. By setting $F_p = 0$ in (7),

$$\tau(X_c - F_c)^+ \qquad \leq \tau \sum_{e=1}^{m} (X_e - F_e)^+$$

$$\begin{aligned}\iff \quad \tau X_c - \tau(X_c - F_c)^+ \quad &\geq \tau \sum_{e=1}^{m} [X_e - (X_e - F_e)^+] \\ \iff \quad \tau \text{min}(X_c, F_c) \quad &\geq \tau \sum_{e=1}^{m} \text{min}(X_e, F_e). \end{aligned} \tag{A53}$$

Thus, the market value of tax benefits of the conglomerate is greater than that of the independent firms:

$$\tau E\left[\beta \text{min}(X_c, F_c)\right] \geq \tau E\left[\beta \sum_{e=1}^{m} \text{min}(X_e, F_e)\right]. \tag{A54}$$

Now, suppose that $\hat{F}_c(\sigma_1^2, \sigma_2^2, ..., \sigma_m^2)$ is the optimal debt of the conglomerate when the degrees of diversification of the independent firm asset portfolios are $\sigma_e^2,\ e = 1, 2, ..., m$. In the case of fully diversified independent firm assets ($\sigma_e^2 = 0,\ e = 1, 2, ..., m$), there exist coefficients $(\bar{a}_e, b_e)$ and a lognormally distributed X, such that the payoff X_e to asset e is given by (24), which implies that

$$X_e = Exp(a_e)X^{b_e},\ e = 1, 2, ..., m,\ \ \bar{a}_e \equiv a_e + ln(X_e(0)) - b_e ln(X(0)), \tag{A55}$$

using (20). Let the independent firms choose debts

$$F_e \equiv Exp(\bar{a}_e)(\bar{F})^{b_e},\ \ e = 1, 2, ..., m, \tag{A56}$$

where $\bar{F}$ is a solution to

$$\hat{F}_c(1) = \sum_{e=1}^{m} Exp(\bar{a}_e)(\bar{F})^{b_e} \equiv g(\bar{F}). \tag{A57}$$

Observe that $b_e > 0$ for all $e = 1, 2, ..., m$, and, hence, $\lim_{\bar{F}\to 0} g(\bar{F}) = 0$ and $\lim_{\bar{F}\to\infty} g(\bar{F}) = \infty$, which means that there exists an $\bar{F}$ such that $g(\bar{F}) = \hat{F}_c(1)$. Similarly, if $b_e < 0$ for all $e = 1, 2, ..., m$, then $\lim_{\bar{F}\to 0} g(\bar{F}) = \infty$ and $\lim_{\bar{F}\to\infty} g(\bar{F}) = 0$, which means that there exists an $\bar{F}$ such that $g(\bar{F}) = \hat{F}_c(1)$. Then (A55)-(A56) imply that

$$J_e = \{X_e < F_e\} \iff \{X < \bar{F}\} \iff \left\{X_c < \sum_{e=1}^{m} F_e\right\} = J_c, \tag{A58}$$

since $\{X_e < F_e\},\ \forall e$ or $\{X_e \geq F_e\},\ \forall e$. Thus, $J_1 = J_2 = ... = J_m = J_c$, i.e., $\Pr(J_1) = \Pr(J_2) = ... \Pr(J_m) = \Pr(J_c)$ and

$$fF_c \Pr(X_c < Fc) = \sum_{e=1}^{m} fF_e \Pr(X_c < F_c) = \sum_{e=1}^{m} fF_e \Pr(X_e < F_e), \tag{A59}$$

which implies that

$$\beta \sum_{e=1}^{m} E[\gamma(X_e, F_e)] < \beta E[\gamma(X_c, F_c)], \tag{A60}$$

because the sum of the DOW values of a set of independent firms is strictly greater than the DOW value of the conglomerate having the same total debt, i.e.,

$$\sum_{e=1}^{m} \beta E(fF_e - X_e)^+ > \beta E(fF_c - X_c)^+, \tag{A61}$$

since (by substituting fF_e for F_e in (A53))

$$\begin{aligned} & \min(X_c, fF_c) && \geq \sum_{e=1}^{m} \min(X_e, fF_e) \\ \Longleftrightarrow \quad & fF_c - \min(X_c, fF_c) && \leq \sum_{e=1}^{m} fF_e - \sum_{e=1}^{m} \min(X_e, fF_e) \\ & (fF_c - X_c)^+ && \leq \sum_{e=1}^{m} (fF_e - X_e)^+. \end{aligned} \tag{A62}$$

Thus, for fully diversified independent firm asset portfolios, the independent firms together can generate as high a market value of tax benefits as the conglomerate, but at a lower market value of the bankruptcy cost (diversification gain and cost) than the conglomerate. That is, for fully diversified [or by continuity slightly less than fully diversified] independent firm asset portfolios, $\mathcal{V}(A2) > \mathcal{V}(A3)$, if $\alpha_e = 0,\ e = 1, 2, ..., m, c$; and, by continuity, there exist $\alpha_e > 0,\ e = 1, 2, ..., m$, with $\sum_{e=1}^{m} \alpha_e > \alpha_c$ such that $\mathcal{V}(A2) > \mathcal{V}(A3)$. (b) Further, for a sufficiently large difference in the formation costs, $\sum_{e=1}^{m} \alpha_e - \alpha_c > 0$, $\mathcal{V}(A2) < \mathcal{V}(A3)$. ∎

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