An Economic Theory of Constitutional Governance

Sankarshan Acharya

University of Illinois at Chicago and

Research Center for Finance and Governance (India)

Email: sacharya@uic.edu*

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Abstract: The economy is modeled as a set of leveraged firms (including households) with potentially superior information who choose their assets to maximize their net-worth, while an efficient, not-for-profit government enacts and administers constitutional rules for free trading of goods, services and assets. In equilibrium, (a) the moral hazard risk stemming from any potentially superior information is efficiently dissipated and (b) the government forms a *Safe Bank* (i) to offer security of the safe asset (deposits), chosen by some firms, to preclude financial panics, and to grant equal privilege to <u>all</u> firms as per constitution. The model proves that eliminating the Federal Deposit Insurance Corporation and amending the Federal Reserve Act to grant equal privilege to all firms (not just financial firms) is constitutional and efficient, which then makes the Federal Reserve identical to the *Safe Bank.* In equilibrium, (c) the asset risk premium is negatively related to volatility of a levered firm, (d) the asset volatility and risk premium are both increasing functions of the asset-to-debt ratio, and (e) the minimum threshold asset-to-debt ratio below which the firm goes bankrupt is an increasing function of the asset risk premium.

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1 Introduction

The axioms used to develop a theory of constitutional governance are as follows:

A1. Firms including individual households seek to operate for profit. Firms' liabilities to their stakeholders are restricted to their assets. If the assets cannot cover liabilities, stakeholders are allocated by the standard priority rule. Any aggrieved stakeholder can appeal to the bankruptcy court for such allocation.
A2. The government seeks to minimize its costs to operate efficiently, but not for profit.
A3. The market is not restricted. It values assets in equilibrium by the standard arbitrage pricing principle. That is, the market is not hobbled either by the government or by the firms to value assets by the arbitrage pricing principle. The arbitrage pricing principle basically determines relative values of various assets, consistent with their risks in equilibrium. Free or unrestricted market means that the government does not intervene, for example, to bail out failing firms, to force mergers among firms, to inject new taxpayer funds to firms or to buy "illiquid" assets of firms. Free trading subsumes arbitrage trading of securities including derivatives and hedging of risk associated with such trading. There is thus no restriction imposed on any enterprise including hedge funds, insurance companies, commercial banks and investment banks. Free trading is modeled via arbitrage pricing of assets.
A4. The constitution grants equality to all firms and government. The government enacts and enforces rules according to the constitution. The constitution, for example, forbids the government or any of its agencies or private group of firms to adopt any rule, procedure or practice to deprive any person of life, liberty, or property; or to deny to any person within its jurisdiction the equal protection of the laws. This is consistent with the Amendment 14 of the U.S. constitution.

A government can run with policies based primarily on maximizing the market values of firms without seeking to reduce the cost to taxpayers and without a free trading policy. This government is unconstitutional because of abandonment of the constitutional provision of free trading in a market economy. Moreover, this government will be inefficient, i.e., neither small nor smart. It will eventually fail. How? Each group of firms will lobby with this government to adopt those policies that maximize their market values. This would imply larger product prices and smaller wages for people. This will raise the short-run profits of firms, leading to higher tax revenues and a larger size of the government. Such short-run policies will, however, be myopic as they will gradually pressure the households to earn lower wages and to support higher prices. Eventually many households will fail to afford the rising prices with their shrinking incomes. This will lead to a failure of many firms, loss of jobs for people and shrinking tax revenues for the government. The economy will then nosedive. The surviving firms with the resources will gain the power to dictate policies, which will allow less freedom to people and lead to a repetition of the same game. A government thus run by policies that exclusively maximize the market values of firms will eventually collapse. This is the danger faced during the depressions. The unconstitutional governance with profit maximizing firms constitutes a *laissez faire* capitalism that prevailed and collapsed during the Great Depression.

A free market economy is enshrined in the U.S. constitution. Freedom of firms and markets from government interference is necessary for innovation and competitiveness. The prevailing economic paradigm has accordingly professed noninterference of government in firms and markets. But this economic paradigm has failed to explain the mass failures of farms and markets witnessed during the Great Depression or the Great Recession. A rational economic paradigm cannot characterize such catastrophes as slapping by the invisible hand (god) to justify government bailouts of failed firms. Acharya (2010) shows that the current paradigm of money and finance causes unconstitutional usurpation and destruction of private capital in time, leading to depressions and recessions.

This paper shows that (a) a substantial reform of the Federal Reserve Act (FRA) of 1913 and (b) an elimination of the Federal Deposit Insurance Corporation created by the Glass-Steagall Act of 1933 are necessary to make the system of money and finance efficient (preserving capital) and constitutional (avoiding unconstitutional usurpation) within a free market economy supported by an efficient not-for-profit government. How? In equilibrium of the model, (a) some firms choose the safest asset (riskless deposits), while others choose risky assets to earn returns consistent with risk, and (b) the constitutional government does not intervene in the markets, but offers a secure custody of riskless deposits in a government entity called *Safe Bank*. The *Safe Bank* obviates financial panics, thus, eliminating the necessity for the federal insurance of deposits via risky financial firms. The FDIC now collects the price of federal deposit insurance from the financial firms who maintain their efficiency by passing on this cost in the form of lower interest on deposits below the equilibrium interest rate, thus making the FDIC the source of inefficient governance. The *Safe Bank* precludes this inefficiency by rendering the FDIC redundant as a panic-prevention agency. In addition, the *Safe Bank* grants the same privilege to all firms to deposit their savings with government security without any ceiling. If current the Federal Reserve Act is amended to grant the same privilege to all firms, then the Federal Reserve will be identical to the *Safe Bank* obtained within the equilibrium of the model.

Specific efficient amendments implied by the economic theory of constitutional governance include the following:

- Abolish the Federal Deposit Insurance Corporation.
- Amend the FRA to let the Federal Reserve grant equal privilege of its secured custody to the riskless deposits of all firms including households without any limit. The amended FRA will, for example, ensure that the Federal Reserve extends the same privilege of securing riskless deposits, currently

available for only the capital reserve accounts of financial firms and foreign central banks, to all firms including households.

- Amend the FRA to let the government borrow directly from the Federal Reserve.

The economic theory of constitutional governance thus proves that the FRA-1913 is unconstitutional. The FRA has been the incipient source of systemic weakness in the U.S. economy.[1] This Act was imposed when the Congress and the President had ceded all their economic powers to the financial institutions that had survived after the banking panics of 1907. If the governments power to enact and administer constitutional acts is voided within the model, the current FRA-FDICA will prevail in equilibrium. But a government with the power to enact and administer constitutional acts within the model makes the current FRA and the FDIC unconstitutional and inefficient as compared to the $Safe\ Bank$:

- The FRA-FDICA forces nonfinancial firms including households to hold their safe deposits in risky financial firms and to receive a government (FDIC) guarantee up to a maximum ceiling, which was raised from $100000 to 250000$ in 2008. The FDIC collects a deposit insurance premium from risky financial firms to recover losses due to such insurance. The risky financial firms efficiently transfer the cost of deposit insurance to depositors by lowering the rate of interest on deposits. The depositors thus receive a lower rate of interest than they that they can get by directly lending the government through the Safe Bank in my model. This makes the FRA-FDICA inefficient and unconstitutional and Safe Bank efficient and constitutional for deposits below the ceiling on federal deposit insurance.

- Savings greater than the FDIC insurance ceiling, deposited in risky financial firms, have no government security like that extended by the FRA to the reserve accounts of financial firms and foreign central banks without limit. Such unequal treatment makes the FRA-FDIC unconstitutional. The unconstitutionality is precluded by the Safe Bank in my model.

- Nonfinancial firms and most small financial firms cannot participate in primary auctions of Treasury securities like a few privileged large financial firms currently enjoy, making the FRA-FDICA unconstitutional and inefficient. The Safe Bank averts such unconstitutionality and inefficiency by granting equal opportunity to all firms to efficiently lend their funds at the same rate directly to the government. The Safe Bank obviates and stops the primary auction of Treasury securities and offers the same efficient privilege of lending the government to all firms.

[1]The details are written by the author in several memos published in the Internet, pro-prosperity.com, and lately articulated to reflect this paper in a memo (dated October 18, 2010) submitted to the President and Congress with a subject, "Correcting Systemic Weakness to Avert Great Depression," which is available on request (sacharya@uic.edu).

The economic theory of constitutional governance implies unconstitutionality and inefficiency of all forms of government subsidies to special interest groups including through government sponsored enterprises like Fannie Mae and Freddie Mac. For example, the recent transfer of massive quantities of worthless toxic mortgage assets from private financial firms to Fannie and Freddie was a form of inefficient and unconstitutional subsidy that the government gave to the private financial firms at a huge cost to the GSE stakeholders including taxpayers. The 10% preferred dividend collected by the Treasury Department from its Fannie and Freddie preferred stocks for money created by the Federal Reserve at no cost is also a form subsidy that penalizes GSE stakeholders and benefits private short-sellers of GSE securities; this is inefficient and unconstitutional. The Security and Exchange Commission's short-selling rule is likewise inefficient and unconstitutional because it permits transfer of wealth to short-sellers from the true security holders through an artificial creation of virtual shares (not issued by the firms) for driving down the security price below the equilibrium level attained with free trading of only the real shares issued by firms.[2]

The model thus offers an efficient and constitutional paradigm to analyze every government economic policy for its efficiency and constitutionality.

When the Financial Crisis Inquiry Commission asked the heads of the Federal Reserve and FDIC to testify under oath on September 2, 2010 about any common policy factor behind their 2008 rescue efforts, it found none. Furthermore, the government regulators had asked many large financial firms to raise billions of dollars of new capital before shutting them down. Such *ad hoc* regulatory response decimated the confidence of investors in risky firms with potential regulatory interference. This has resulted in massive flows of funds to Treasury bonds. The firms that innovate and create jobs are now facing credit crunch, causing severe unemployment and uncertainty in the economy. The *Safe Bank* in the model averts financial panics and eliminates government insurance or protection of financial firms, which precludes any form of government intervention in firms or markets, thus stripping the government regulators of their power to choose, *ad hoc*, the firms to rescue or shut down in a panic-driven crisis.

When the panic spread to cause a run on the $3 trillion money market funds in 2008, the regulators too must have panicked to guarantee the money market funds and bank debts and to significantly raise the deposit insurance limit. This amounted to the government effectively providing the security of a *Safe Bank* to everyone. If the *Safe Bank* is formally established by suitably amending the Federal Reserve Act and eliminating the federal deposit insurance, there will be no panic in the financial markets because every firm will have the secured saving facility without a ceiling, the regulatory agencies will have no scope for *ad hoc* responses, and the investors will trust firms and markets since there will be little fear of *ad hoc* regulatory intervention.

The prevailing paradigm advocates no government intervention in firms as long as the mega players

[2]See Acharya, S. (2007) and Acharya (2010)

privatize profits by piling risk on taxpayers and households. But it schizophrenically invokes the invisible hand (god) to explain the mass failure of firms to justify government bailouts.[3] In the theory presented here, the government does not interfere in firms, ex ante as well as ex post, according to constitution and financial panics are precluded.

The common ulterior longing of the people seems to be a small but smart government. The efficient governance presented in this paper is consistent with such common longing. It is a novel economic paradigm in which a not-for-profit government sets policies by minimizing the cost to taxpayers of maintaining a constitutionally mandated free trading, market economy while letting firms maximize their market values.

Owners of firms in the model may have potentially superior information while choosing their assets and the debt holders may be thus disadvantaged. Observe that when households operate like firms with borrowed funds to buy homes and other merchandize, the debtholders (lenders) may be less informed about choices of borrowers. The not-for-profit government lets the debt holders exercise their rights under constitution within the model. An arbitrage-free equilibrium in this model is defined as a set of choices by the players (debt and equity holders of firms and the government) and the market prices of assets to preclude arbitrage (unlimited) profit opportunities. The following are the other results in an arbitrage-free equilibrium:

1. Free trading, not government intervention, is necessary to discover and price asset risk.
2. Free trading dissipates the moral hazard risk to taxpayers stemming from any potentially superior information that a firm may have in choosing assets.
3. Free trading begets the first-best (efficient) resolution of moral hazard in equilibrium.
4. The asset risk premium is negatively related to volatility of a levered firm.
5. The asset volatility and risk premium are both increasing functions of the asset-to-debt ratio.
6. The minimum threshold asset-to-debt ratio below which the firm goes bankrupt is an increasing function of the asset risk premium.

[3]I started searching for the truth behind financial catastrophes in the aftermath of the 1987 market crash. I have been so deeply embedded in the game as a researcher since then and so determined to avert any future crash that I could vividly see how mega market players, not invisible hands nor gods, were financially slapping the rest to create yet another depression for self-enrichment and entrenchment in power: http://pro-prosperity.com/Global%20Economy%20Chatterbox/Warning-USCongress-In-2003-On-Home-Mortgage-Debacle.html. The specific shenanigans that led to massive failure of many name-brand financial firms are presented in Acharya (2003) and Acharya (2010).

7. It is an efficient (first-best) policy (i) to discontinue the (current) government insurance or regulation or intervention of financial firms and (ii) to create a (new) government owned central banking facility called *Safe Bank* in which all firms (financial and non-financial) can deposit their savings without inefficiently being forced to save their deposits in risky financial firms insured partly or fully by the government.

The events during the recent financial meltdown in 2008 corroborate the first two results: the risks of various securities in the market place could neither be ascertained nor priced because the government did not allow free trading of mega banks that failed. The government chose to let some banks fail, while bailing out other banks with no uniform policy. The government injected massive sums of new money to banks, in addition to the Federal Reserve buying the "illiquid" bank assets. The government did not let free trading of the "illiquid" bank assets and rather bought them at fictitious prices set by bureaucrats, as opposed to prices determined in free markets as per constitution. The government actually bailed out some firms while letting others including most households fail depending on preference of decision makers, not based on the principle of free markets and trading. If free trading were permitted, the risks of various securities could have been ascertained and priced by the markets, consistent with the first two results of the model.

The model does not assume that the equity holders necessarily know more than the market or that they are just as informed as the market about the risk of assets they choose. The model realistically assumes, however, that the equity holders in either case choose the asset composition (volatility risk) as owners of the firm. It is then proved that the equilibrium outcome in the case the equity holders may know more than the market is the same as that obtained in the other case when they know as much as the market. This proves that the equilibrium is indeed efficient or first-best.

Free trading (not government intervention) is necessary to obtain equilibrium within the model in either the symmetric or the asymmetric information case. Without free trading, the market breaks down because it can neither discover risk nor determine prices of assets within the model. This is very consistent with the events of 2008.

The potential informational advantage of the equity holders does not impose moral hazard risk or cost on taxpayers as long as free trading is permitted within the model. Free trading is thus crucial for efficient resolution of moral hazard within the model.

The economic paradigm presents here an alternative theory of smart governance operating at the lowest cost to taxpayers within a constitutionally mandated free market economy. The equilibrium policies implied by this theory are constitutional. Theoretically, these policies avoid the pitfalls of *laissez faire* capitalism and thereby imply long-run prosperity amid stability by averting continual failures of firms and by raising incomes, savings and employment of people. This theory thus presents a novel platform to pursue policies by a not-for-profit, constitutional government with an objective to minimize the cost to taxpayers. This platform is fundamentally important for a nation with a constitutionally mandated market economy and free trading. This platform for governance and the equilibrium policies within it are profoundly important for fulfilling the common longing of people.

A main implication of this theory is efficient resolution of moral hazard through first-best equilibrium policies. Moral hazard in banking stems from the explicit government guarantee of bank deposits and implicit protection of too-big-to fail banks. The Financial Regulatory Bill passed in July 2010 promises to not protect any bank, no matter how large it is. But this promise is impossible to keep when the FDIC goal of minimizing losses to its insurance fund is systemically leading to acquisition of smaller problem banks by the biggest, leading the latter to control most bank assets. Bank executives tend to use the guaranteed funds on highly risky bets. If the bets succeed in the short-run, they usurp the profits as bonuses. If the bets fail, they seek injection of new money from taxpayers. Taxpayers are forced to bail out the gambling banks which are too big to fail, lest a severe financial meltdown will ensue to depress the economy and households. The US Congress instituted the federal deposit guarantees to prevent banking panics and runs and a recurrence of the Great Depression in 1933. But the U.S. faced similar panics with massive withdrawals from money market funds and a catastrophic failure of many major banks in 2008, despite all the guarantees. Pension plans and retirement funds lost trillions of dollars when highly leveraged banks had to sell their assets to generate the needed capitals for their survival.

Here is an illustration of how the bank holding company structure was used in highly leveraged bets with insured debt and deposits:

An Example: *A bank is formed with \$8 of equity capital and \$92 of insured deposits and debts. This makes \$100 for investment in assets like mortgage loans, business loans, stocks, bonds and derivatives. This bank meets the criteria for minimum capital of 8% under the FDICIA 1991. If the return on this banks investment is 6% and the cost of funds is 4%, the bank earns a net \$2 or 2% on assets and 25% on equity per year. The owner gets his equity back in 4 years and continues to own his bank freely by meeting the FDICIA 1991 minimum capital requirements.*

But the owner is not satiated with a mere 25% return on equity investment. He then forms a one-bank bank holding company (BHC) comprising a parent company (PC) and a subsidiary bank (SB). The SB is legally firewalled from the PC so that the SB can fail but not the PC. The previous bank is now restructured with the \$8 of equity and \$92 of debt held by the PC. This makes the total PC assets of \$100. The PC assets are now injected as equity to the SB. The SB then raises \$900 in additional debt and insured deposits. The PC now has 8% capital-to-assets ratio and the SB has 10% capital-to-assets ratio. Both the PC and the SB are permitted under the FDICIA 1991 because each entity meets the minimum capital requirement of 8%. The BHC now has \$1000 in total assets with a net income of 2% or \$20 which makes $20/8 = 2.5$ or 250% return per year on the equity investment of \$8. This is a highly profitable real strategy for the owner of a bank.

The same bank, after becoming a BHC, now has the same \$8 in equity, but \$992 in debt and insured deposits. This works out to a mere 0.8% (8/1000) of capital-to-assets ratio on a consolidated basis. Multi-leveraging thus undermines the FDICIA 1991 badly.

This is a real episode of moral hazard by the banks and their regulators: Banks reap enormous wealth through transfers from taxpayers, global depositors and the effective producers of globally competitive goods, services and ideas. They thus have sufficient resources to make hefty political contributions to tinker policies in their favor for continued self-aggrandizement and to offer lucrative positions to bank regulators who permit effective subversion of minimum capital standards. The risky gambles of major bank holding companies - in subprime mortgage debt, derivatives, commodities and global equities - with little capital was bared in 2008 by their failure to raise sufficient capital freshly required to meet the FDICIA minimum on a consolidated basis.

The U.S. Congress must have acted seriously in early 2008 to ensure that bank holdings companies meet the FDICIA 1991 capital standards on a consolidated basis. One can infer this from the vocal public pressure in early 2008 that the Treasury Department exerted on banks to deleverage and raise capitals. The highly leveraged banks that could not raise sufficient capitals fell during 2008. The U.S. Treasury and Federal Reserve have then injected massive sums of money to stabilize the banking institutions. The central bank interest rate has remained at historically low levels. Yet, the unemployment rate has reached higher to 10 percent. The unemployment among males aged 20 to 54 is as high as 20 percent, according to media reports. This is close to the unemployment rate of 25 percent during the Great Depression. The economy may be still at the brink of another Great Depression which may become global this time.

The above example illustrates that moral hazard enormously raises the costs to taxpayers. Moral hazard, if not thwarted, can lead to a thorough failure of governance. A complete failure of the U.S. government can happen if the dollar loses its reserve currency status, i.e., when the rest of the world curtails trading in dollar. But policies to eliminate moral hazard efficiently can prevent this from happening, especially when the rest of the world does not want a demise of the dollar.

In the model, the *Safe Bank* policy, presented above and detailed later, is the unique first-best (efficient) equilibrium resolution of moral hazard. Because the government does not guarantee deposits or firms in this model, there will be no moral hazard risk to taxpayers. The *Safe Bank* will provide the safety to panic prone firms including households. The other firms willing to take risk can invest in uninsured financial firms, which may operate like the current universal banks. Once federal guarantee of deposits is eliminated, only those universal banks that can be highly capitalized will attract the trust and confidence of firms. The *Safe Bank* policy, once enacted, will thus result in highly capitalized universal banks to serve as repertoires of wealth and trust of firms including households willing to take some risk.

The prevailing principle or paradigm of market discipline argues that the market can discipline itself. It advocates freedom in the markets. My theory also advocates free trading and free markets. The prevailing paradigm wants no government intervention in business enterprises, so does my theory. So what is the difference in my model that yields an efficient resolution of moral hazard, namely, a *Safe Bank* in equilibrium? The difference is the role in my model of a not-for-profit government that minimizes the cost to taxpayers and enforces debt and arbitrage pricing contracts. In my model, the government does not intervene ex ante or ex post. The market discipline paradigm seeks no ex ante government intervention while the firms keep privatizing their profits. The market practitioners of this paradigm seek intervention to bail out failing firms by transferring taxpayer funds and thus socialize their losses. The market discipline paradigm does not entertain government only ex ante, but begs the government to bail out failing firms, ex post. The market discipline paradigm is thus schizophrenic. It can survive philosophically only if it conjures the financial depressions, wrought by mass failures of firms, as acts of god or slaps by invisible hands. Only by portraying the financial calamities as acts of god, can it hope to induce *We the People* to print new money, continually.

Money injected to the privileged large financial firms is not in terms of visible bailouts like the Troubled Asset Relief Program of 2008. The Federal Reserve Act permits unconstitutional usurpation of public resources to guarantee replenishment of depleted capital at the biggest privileged financial firms. This is done with (a) the Federal Reserve lowering its interest to lend vast sums of artificially created money to these privileged financial firms and to force the depositors to accept lower interest rates from these firms, and (b) the Treasury borrowing these funds at a significantly higher rate. The spread between the Treasurys higher borrowing rate and the Federal Reserves lower lending rate generates a significant profit for the privileged financial firms to replenish their depleted capitals to reach the required minimum. This unconstitutional system perpetuates an inefficient method of lavishing the bank executives and associated government facilitators with exorbitant pays and perquisites.

In the paradigm presented here, the government does not intervene firms or markets ex ante or ex post. It is, therefore, not schizophrenic. This paradigm is thus robust. It fosters constitutionally mandated discipline among all market players (firms, households and government). What it yields is seriously important: a free market resolution of failing firms without any direct injection of taxpayer funds or indirect transfers to privileged financial firms via the spread between a lower Federal Reserve lending rate and a higher government borrowing rate. Such injections to bail out privileged financial firms is the unconstitutionally practiced system of moral hazard that can be obviated only by reforming the Federal Reserve Act and eliminating the FDIC. The financial regulatory bill passed in July 2010 promises to not save any floundering financial firm, irrespective of its size. But this promise will be hard to keep because this Act preserves the FRA and the FDIC, which seeks to minimize its deposit insurance cost by letting the smaller troubled banks be acquired by the biggest ones over time.

A continuous time dynamic model of a levered firm is considered: Debt-holders incur costs to monitor the firm, periodically. The firm incurs transaction costs to raise new equity. The market acts as a super monitor for valuation of assets to preclude arbitrage opportunities in equilibrium. The government passively enforces such a role for the market to operate freely according to the constitutional axioms states earlier. The government also oversees that debt and pricing contracts are honored as stipulated, for example, via bankruptcy courts. Equity-holders are assumed to maximize the market value of equity and control the choice of the standard deviation (volatility) of the return to assets of the firm.[4]

[4]Within our model, controlling the drift does not make our results different.

Standard asset pricing models imply a linear relationship between the expected return and systematic risk of all efficient portfolios, and show that the expected return to an asset is linearly related to its systematic risk in a frictionless world, where investors prefer a higher mean return, the higher the systematic risk.[5] But these models show no relationship between a real asset's expected return and volatility. We consider an arbitrage-free market with potential (i) incompleteness due to non-uniqueness of the pricing measure and (ii) failure due to non-existence of a solution. We show the existence of a unique equivalent martingale pricing measure in equilibrium of a game between equity-holders and debt-holders, when the market effectively acts as the super monitor and the government facilitates enforcements of contracts as per constitution, in two cases, defined by whether or not the volatility is *ex ante* observable. We show that the expected return-volatility frontiers are the same, whether or not volatility choices are *ex ante* observable. These frontiers result from a dynamic model of the firm with endogenously evolving asset-to-debt ratios. They apply to firms with different asset-to-debt ratios in a cross-section as well as to a firm with asset-to-debt ratios changing over time. The equilibrium state pricing density (Arrow-Debreu prices per unit probability) depends, dynamically, on the asset-to-debt ratio of the firm, setting the asset risk-premium as a function of a firm's asset volatility and asset-to-debt ratio.

An important point that emerges from our analysis is that the risk premium on a levered asset, which must be monitored, depends endogenously on the monitoring process and on the choices of equity-holders and debt-holders, which depend on the leverage of the firm. We thus obtain a theory of evolution of market prices of levered (collateralized) assets. Our endogenous levered asset risk premiums and volatility contrast the He-Leeland (1993) restriction on asset value processes, when a representative agent, investing in assets, maximizes his utility of wealth.[6] The noise trading literature shows that the stock price volatility increases, above its fundamental level, with the degree of noise in trading (Foster and Vishwanathan (1993)), whereas our result implies an evolution of the fundamental volatility of levered assets.

The result that the risk premium and the volatility are negatively related in an arbitrage-free equilibrium seems very consistent with empirical findings of Glosten, Jagannathan and Runkle (1993).[7] There are additional interesting testable implications of our theory. The

[5]Systematic risk refers to the market beta as in the Capital Asset Pricing Model (see, e.g., Sharpe (1964), Lintner (1965), and Mossin (1966)) or a vector of factor betas as in an Arbitrage Pricing Theory (see, e.g., Ross (1976) and Connor (1984)). Fama and French (1992) find that the market beta explains very little of the expected return to stocks.

[6]Merton (1973) and Breeden (1979) develop inter-temporal capital asset pricing models, which restrict the expected instantaneous returns of individual assets relative to the expected instantaneous return of a market portfolio. Rubinstein (1976), Breeden and Litzenberger (1979) and Brennan (1979) have established a consistency between the lognormal process for the market portfolio and a representative investor having constant relative risk-aversion. See also Gennotte and Marsh (1993), where the price process is no longer lognormal if the volatility of the dividend process is random.

[7]Other empirical studies that find a negative relationships between the expected return and the variance include Fama and Schwert (1977), Campbell (1987), Pagan and Hong (1991), Breen, Glosten and Jagannathan (1989), Turner, Starz and Nelson (1989), and Nelson (1991). French, Schwert and Stambaugh (1987), and Campbell and Hentschel (1992) find a positive conditional relationship between expected return and variance. Chan, Karolyi and Stulz (1992) find no variance effect for the US, but find one for the world market portfolio. Support for zero or a positive expected return-variance relationship is mostly based on the standard GARCH-M model. Jagannathan, Glosten and Runkle (1993) argue that this model may not be rich enough and use a more general specification of the GARCH-M model.

result that the asset volatility increases with the asset-to-debt ratio of a firm, given the expected return, also applies to households, who have levered up their investment portfolio. As a household becomes wealthier (indicated by the market value of equity), its assets become more volatile in an arbitrage-free economy. This result does not depend on any specific risk-preference, except that more wealth is preferred to less. The economic intuition for this result is that the greater the wealth (market value of equity) of a firm or household, the less the intensity of monitoring by lenders and the less the cost of raising new funds. This implies that the effective drift rate of the asset values increases in the assets/debt ratio, given volatility, and the market sets the difference between the effective drift rate and the spot interest rate as the risk premium. If the asset-to-debt ratio is increased, the effective drift rate will thus increase, unless the volatility is increased enough to raise the expected monitoring cost and hence to reduce the effective drift rate. Thus, the volatility increases with the asset-to-debt ratio, given the risk premium.

The result that the expected return to an asset increases with the asset-to-debt ratio, given volatility, seems to be economically very interesting and significant. It contrasts the corporate finance text-book wisdom, based on the seminal Modigliani and Miller (1958) propositions and subsequent works by Stiglitz (1974), which imply that the expected return to assets is independent of leverage, although the *cost of equity* and leverage are positively related.[8] Our model also implies that a new equity offering is a predictable function of the asset-to-debt ratio, as we abstract away from the adverse selection or signaling effect of new equity issuance.

Our result that the asset risk premium is an increasing function of the asset-to-debt ratio, can be tested by studying the stock price effect of those equity offerings that can be predicted by pre-determined information.[9]

Another interesting testable implication of our theory is that the equilibrium threshold asset-to-debt ratio below which a firm is closed due to a bankruptcy or a liquidation is an increasing function of the asset risk premium. This means that lenders will optimally impose, as covenants, higher minimum asset-to-debt thresholds for operation of financial and nonfinancial firms with higher risk premiums. Regulators acting in behalf of lenders will likewise impose, optimally, higher minimum asset-to-debt thresholds for operation of regulated financial firms (banks and insurance companies) facing greater market required risk premiums. The world that adopts covenants optimally will see surviving firms with asset-to-debt ratios increasing with their systematic risks. Our result also implies that the systematic risks of bankrupt firms are higher than that of survivors, everything else equal.

Observe that a negative relationship between the common stock expected return and volatility is not completely isomorphic with a firm's asset volatility-risk premium relationship because of the volatility of debt.

[8]Acharya (1994) empirically finds a very robust positively sloped non-linear relationship between the expected rate of return to assets and the equity/asset ratio of a firm, using annual data on all Compustat firms during 1973-1992, controlling for firm-specific attributes.

[9]Acharya (1994) finds that the stock price effect of a predictable equity offering is statistically and economically significantly positive, while that of an unpredictable equity offering is negative using methodologies in Acharya (1993). Thus, the data are consistent with the extant literature on the negative stock price effect of equity offer announcements, as well as with the theory developed here.

That our arbitrage-free equilibrium results are first-best seems very pertinent to an important goal of the economic theory of seeking market-mediated outcomes and examining if they are efficient and attainable. The standard principal-agent theory has relied on *non-market*, contractual arrangements to motivate agents for acting in the interest of less informed principals.[10] Indeed, formulating appropriate incentive contracts even within a non-market setting has not been easy (see, e.g., Milgrom (1981) and Grossman and Hart (1983)). Investigating the role of a capital market to resolve moral hazard associated with a principal-agent model thus appears to be an important contribution of our paper to the literature. The economic intuition for a first-best arbitrage-free equilibrium is that, by using an equivalent pricing measure, the market effectively acts as a super monitor, rewarding the agent (equity-holders) according to the expected return-volatility frontier, while generating an expected rate of return on debt (after accounting for the monitoring cost), which is consistent with the systematic risk of debt. An *ex ante* observable volatility does not matter, because in our model equity-holders realize a lower expected payoff and hence a lower expected return, the higher the volatility. We show that this makes the equity-holders indifferent between (i) running the levered firm and (ii) investing the proceeds from a liquidation of the firm elsewhere in the market. Our approach contrasts, however, Fama's (1980) idea of competition in the managerial labor market, in which a mechanism for preclusion of arbitrage opportunities from trading in managerial product of labor needs to be defined (cf. Holmstrom (1982)). Observe that even the minimized monitoring and equity issuance costs are non-zero in our model, but the market accounts for these costs by appropriately adjusting the state pricing density, resulting in the same value of assets in a world where monitoring is necessary as in a world where it is not necessary.

We present our model of a levered firm in an arbitrage-free economy in Section 2, examine the existence of arbitrage-free equilibriums in Section 3, describe the constitutional system of governance of banks and markets that emerges in equilibrium in Section 4, and conclude in Section 5.

[10] See, e.g., Ross (1973), Harris and Raviv (1979), Holmstrom (1979), Shavell (1979), Holmstrom (1982), Grossman and Hart (1983), Holmstrom and Milgrom (1987), Williams (1987), and John (1987). Jensen and Meckling (1976) offer many insights into the theory of agency of a corporate firm. Fama (1980) argues about competition in the *managerial labor market* to resolve incentive problems, especially between equity-holders and managers of large corporations. A recent work (Acharya (1992)) shows that equity-holders maximize the market value of their firm by optimally timing replacement of managers.

2. Efficiency Due to Arbitrage Pricing of A Levered Firm

Consider a representative firm with risky debt and equity over a discrete sequence of time $\{t = 0, 1, 2, ..., T\}$, where T is the date of closure of the firm due to either bankruptcy or liquidation described later. The equity-holders may raise new equity capital X_t on date t at a new capital issuance transaction cost of $W_t \equiv W_t(X_t) \geq 0$. A negative X_t is interpreted as a divided payout, when the transaction cost is zero. Let V_t denote the pre-equity issuance true value of assets of the firm at t, and F the firm's debt obligation, which is assumed to be fixed on all dates.[11] We abstract away from the determination of the level of debt. Given F, the leverage ratios of the firm evolve endogenously as $F/(V_t + X_t)$. If the firm is not liquidated on any date t, the debt-holders are paid a coupon amount, $C_t \leq X_t + V_t$. Conditional on liquidation, equity-holders receive $(V_t - F)^+ \equiv \max(0, V_t - F)$ and debt-holders receive $\min(F, V_t)$, where, for any variable ϵ, $(\epsilon)^+ = \epsilon$ if and only if $\epsilon > 0$ and 0 otherwise.

If the firm invests its net assets $(V_t + X_t - C_t)$ at t, it generates a payoff V_{t+1} at $t + 1$, given by

$$V_{t+1} \equiv V_{t+1}(V_t + X_t - C_t, \sigma_t) \equiv \tilde{V}_t(t+1), \tag{1}$$

where $\{\tilde{V}_t(\tau),\ \tau \in [t, t+1]\}$ is assumed to be generated by a Brownian motion process,[12]

$$\tilde{V}_t(\tau) = (V_t + X_t - C_t)Exp\left(\int_t^\tau \mu_t(u)du - \frac{1}{2}\int_t^\tau \sigma_t^2(u)du + \int_t^\tau \sigma_t(u)dZ(u)\right), \tag{2}$$

where dZ is a standard Brownian motion, which generates a filtration $\{\mathcal{F}_\tau\}$ in a complete probability space $(\Omega, \mathcal{F}, P)$ with a the probability measure P, set of all states Ω and $\mathcal{F} = \mathcal{F}_T$. A Brownian motion process is equivalent to normal distribution for the continuously compounded instantaneous rate of return on assets, which is consistent with assumptions made in the literature. In equation (2), $\mu_t(u)$ is the instantaneous expected rate of return to assets, and $\sigma_t(u) \in \Sigma \equiv (\underline{\sigma}, \overline{\sigma})$ is the asset volatility (standard deviation of the instantaneous rate of return) at time u. The subscript t denotes that the drift and the volatility processes can change at time t. We thus lay the foundation of a constitutionally scripted market economy to allow free trading of failing firms at prices determined by market participants within our model. This is significant when the government does not (i) run for profit or (ii) grant especial favor to some firms while penalizing others, but lets the debtholders of firms preserve their rights under the law. This leads to a determination of optimal policies for governance of firms without intervention in a free market economy. This is developed after obtaining the equilibrium results. The technical nature of the model is consistent with the

[11] A fixed debt level is not crucial for our results, but it avoids the unnecessary specifications for determining the level of debt, endogenously. Further, a specification that the debt obligations follow a deterministic or stochastic process is a trivial extension of our model. A stochastic process for debt obligations will involve option valuation techniques involving stochastic exercise prices (see, e.g., Amin and Jarrow (1992), Heath, Jarrow and Morton (1992)).

[12] Note that we do not model for regular dividend payouts by the firm. If the firm pays a regular dividend, μ_t will be adjusted for a continuously compounded rate of the payout. A negative X_t can be interpreted as a special (incremental) dividend.

reality that money grows continuously while decisions by agents are made at discrete points in time.

We assume that the market and the debt-holders can observe the amount of new capital raised X_t at no cost. But they have to incur a monitoring cost to observe or assess the true value of assets V_t and the expected return on assets μ_t. We denote the monitoring cost by $M_t \equiv M_t(V_t, F) \in (0, \bar{M})$, where $\bar{M} < F$. We assume that the assets chosen or invented by the owners of the firm generate expected rates of returns (the sequence of $\{\mu_t\}$) which are exogenous to the model.

We endogenously determine the volatility of assets, $\{\sigma_t\}$, by maximizing the market value of equity in ***two cases,*** identified by whether or not equity-holders' choices of $\{\sigma_t\}$ are *ex ante* observable by the debt-holders and the market. The rime series sequence of chosen volatility $[\{\sigma_t(u) \in [t, t+1]\}]$ is simply denoted by σ_t. To avoid triviality, choices of volatility are independent of boundary values $\underline{\sigma}$ and $\overline{\sigma}$ which are assumed to be common knowledge.

Debt-holders face moral hazard when the risk due to the firm's choice of volatility σ_t is *ex ante* unobservable, because their payoff at $t+1$ is affected by risk σ_t. Debt-holders can, however, incur a monitoring cost M_t to observe at (V_t, μ_t), whether or not they face moral hazard due to the firm's choice of σ_t.[13]

We assume the existence of a locally riskless discount factor

$$\tilde{\beta}_t(\tau) = Exp\left(-\int_t^{\tau} r(u)du\right), \tag{3}$$

where $r(u)$ is the instantaneous yield (spot rate) on a continuously rolled over safe asset (money market account) at time u. For simplicity of notations, we define the periodic discount factors by $\beta_t = \tilde{\beta}_t(t+1)$, which is the value at t of a pure discount bond paying \$1 at $t+1$. The excess expected instantaneous return on an investment in assets of the firm, $\mu_t(u) - r(u)$, and the volatility, $\sigma_t(u)$, are assumed to be adapted to $\mathcal{F}_u$ and jointly measurable and uniformly bounded on $[t, t+1] \times \Omega$. This means that these parameters are well defined.

The original probability measure can be used to determine the expected value of random future payoff to an asset, which can be discounted by the market required risk-adjusted cost of capital (equal to riskless rate plus risk premium) to obtain the current fair market value of the asset. This standard valuation procedure is infeasible for derivatives. Derivative assets are, therefore, priced by arbitrage.

The arbitrage pricing procedure constructs a pricing measure which is equivalent to the original probability measure, P. The equivalent measure is used to determine a weighted

[13] The literature on financial intermediation, e.g., Williamson (1986) shows that if lenders monitor a borrowing firm only when the firm defaults and if the cost of such *ex post* monitoring is positive, the firm will voluntarily disclose its true payoffs whenever it defaults. [See also Boyd and Prescott (1986), Diamond (1984), Ramakrishnan and Thakor (1984), and Williamson (1986).] Thus, the other ''moral hazard'' due to the possibility disclosing different information than the true value of assets (payoffs) on each date is resolved within our model due to our costly *ex post* monitoring. Our debt-holders monitor the firm, even when the firm does not default, because of the moral hazard due to the asset volatility choice and to provide the market with a credible information about (V_t, μ_t). If debt-holders monitored only when the firm defaulted, firms would not necessarily disclose their payoffs when they did not default, which would not solve our moral hazard problem.

value of the random future payoffs to the asset. The weighted value is known as the certainty equivalent of the expected payoff to the asset. The certainty equivalent so obtained can be simply discounted by the riskless rate to find the current fair market value of the asset. If assets trade at their fair market values so determined, no arbitrage opportunities exist, i.e., no non-positive current investment in an asset can yield a positive future value.

A frictionless setting within our model will result when the costs of monitoring and raising capital are zero or $M_t = W_t = 0$. Then the equivalent pricing measure will be determined in the standard way [see, e.g., Harrison and Kreps (1979) and Harrison and Pliska (1983)]. In this case, the instantaneous risk premium on the asset under the original probability measure P is the drift of the discounted asset values $(\tilde{\beta}_t(u)\tilde{V}_t(u))$ equal to $\mu_t(u) - r(u)$ at time u. The transformation under the equivalent pricing measure results in certainty equivalents of these discounted asset values $(\tilde{\beta}_t(u)\tilde{V}_t(u)\xi_t(u))$ with no drift. Whenever such a transformation exists, $\nu_t(u) \equiv \mu_t(u) - r(u)$ is the instantaneous risk-premium on the asset.[14] The leverage, volatility and drift choices are irrelevant to the market value maximizing equity-holders in a frictionless market, consistent with the Modigliani-Miller theorem.

A market with friction gives rise to several critical and interesting problems. Even if the volatility risk, σ_t, is observable at no cost (moral hazard is absent), assets cannot be traded when V_t and μ_t are unobservable. If, however, the firm's debt and equity values can be determined in some equilibrium, an indirect trading on assets is feasible. But this involves transaction and monitoring costs of observing (V_t, μ_t), whether or not there is moral hazard. The risk premium, ν_t, must be so determined that the market could continue to price assets with no arbitrage. The following proposition shows that, whether or not moral hazard is present, the effective expected rate of return on assets (drift) will be potentially lower if the transaction and monitoring costs are nontrivial than otherwise.

Proposition 1: Suppose that (i) the firm is not bankrupt or liquidated at t, (ii) the equity-holders raise new capital X_t by incurring a cost $W_t(X_t)$, the debt-holders are paid a coupon amount $C_t \leq V_t + X_t$, (iii) the firm invests its net assets $(V_t + X_t - C_t)$ in its technology with state variables (V_t, μ_t, σ_t) costlessly observed at t, (iv) the debt-holders observe the payoff $V_{t+1}(V_t + X_t - C_t, \sigma_t)$ only by incurring a cost $M_{t+1}(V_{t+1})$ at $t+1$, (v) the firm is liquidated at $t+1$, and (vi) an equivalent martingale pricing measure Q^ν exists. Then, under the original probability measure P, the risk premium on investment in debt and equity of the firm, $\nu_t(u)$, is less than the risk premium of $\mu_t(u) - r(u)$ which could be feasible in a frictionless (costless) world.

Proposition 1 does not characterize any equilibrium. At this stage of the paper, it motivates an important point that the costs associated with frictions like monitoring and capital issuance can effectively reduce the risk premium, denoted as ν_t. This risk premium is determined later in the two cases: (i) moral hazard defined by a situation in which volatility σ_t is not observable *ex ante*, and (ii) no moral hazard when σ_t can be observed *ex ante*. The

[14]Observe that we have not specified the cost to the initial owners of inventing $\{\mu_t\}$, and hence not ruled out a positive net present value, which is the market value at $t = 0$ minus the initial cost of invention of the technology.

main point made by Proposition 1 is that the market determines the pricing measure by accounting for the monitoring and capital issuance costs as well as for the rational behavior of equity-holders and debt-holders.

Proposition 1 shows a reduction in the risk premium due to the costs associated with friction. A lower risk premium implies a lower market perceived risk. This proposition thus reflects the real world where the information generated from monitoring and capital issuance tends to refine the uncertainty and thus reduce the risk perceived to be associated with the firm.

Proposition 1 also shows that the market sufficiently raises the current value of the future payoff to an asset by framing the equivalent pricing measure appropriately in order to compensate for the costs associated with friction.

Proposition 1 proves that the current market value of an investment in the assets of the firm facing friction or no friction is the same. Only the risk premium is adjusted through a refinement of market perceived risk due to monitoring and capital issuance. The adjustment is achieved via an appropriate equivalent pricing measure and the associated state prices that account for the costs of friction.

Practical Implications of Proposition 1: This proposition shows that higher monitoring and new capital issuance costs imply lower expected returns on assets of a firm. This can be practically used for trading in the real world: Form two ratios for each publicly traded company using quarterly time series data; monitoring (auditing) cost divided by the market value of assets and new capital issuance cost divided by the amount of capital raised. Take the difference between either firm-specific ratio and the average of the corresponding ratio over all firms in the dataset. These differences represent the excess firm-specific costs. Use these differences as two independent factors to estimate the risk weights (slope coefficients) in a cross-sectional time series regression of the market model with the quarterly returns on assets of firms as the dependent variable. Proposition 1 implies that these risk-weights will be negative. If the actual market price of assets deviates significantly from that implied by these risk weights, then one can trade in equity and debt of the firm to generate profits.

Clearly, in a world with friction, monitoring plays an important role in revealing the true state of the firm defined by the current asset value and expected return (V_t, μ_t). If these states can be observed at no cost, the total market value of an investment in the firm could be incrementally higher than otherwise by an amount equal to $(V_t + X_t - C_t)\,(R_t - 1)$, where

$$R_t \equiv e^{\left(\int_t^{t+1}[\mu_t(u)-r(u)-\nu_t(u)]du\right)}$$

is the excess expected return on assets over time period $(t, t+1)$ for $0 \le \nu_t(u) \le \mu_t(u) - r(u)$ and $t \le u \le t+1$. Note that $(R_t - 1)$ is the incremental value per dollar invested in the assets of the firm and $(V_t + X_t - C_t)$ is the net investment in assets carried forward from time t to $t+1$. The incremental market value improvement due to a costless revelation of the state of the firm cannot realistically exceed, however, the maximum possible total cost due of monitoring and new capital issuance, which is formally stated as an assumption in the model.

Assumption 1: $(V_t + X_t - C_t)\,[R_t - 1] \leq \beta_t \bar{M} + W_t(X_t)$.

If the risk premium ($\nu_t(u)$) were given, one could assume the effective expected return to assets in (2) as equal to $\nu_t(u) + r(u)$ at time u. In general, the risk premium would depend on stakeholders' choices for new capital, coupon and asset volatility (X_t, C_t, σ_t) as well as on the equity-holder-debt-holder conflict. In particular, the risk premium ($\nu_t(u)$) could be related to the firm's leverage via its dependence on the value of asset carried forward ($V_t + X_t - C_t$) and outstanding debt (F). This contrasts the pure Modigliani-Miller result in a frictionless world that the expected rate of return to assets is independent of leverage and dividend payout. Negative new capital within our model is divided payout.

Proposition 1 does not show how stakeholders choose their strategies or the nature of dependence of the risk premium ($\nu_t(u)$) on the chosen capital issuance, coupon rate and volatility. A sequence of volatility choices $\{\sigma_t\}$ may not exist, and, if one exists, it may not result in a unique set of state prices. The non-uniqueness is a consequence of market incompleteness and the non-existence indicates a market failure. In the case of non-uniqueness, each set of state prices could result in non-unique values of the derivative securities (debt and equity), which are claims on (derived from) the underlying firm asset values (V_t).

Equity-holder-Debt-holder Game when Market Values Securities by Arbitrage

One main contribution of this paper is showing the existence of a unique set of state prices, which define volatility-risk premium frontiers that are dynamically dependent on V_t in equilibrium of a ***game*** among three parties: the market, equity-holders and debt-holders, where the equity-holders control the choice of volatility σ_t as well as new capital X_t. The government is a passive enforcer of constitutional rules defined by the axioms A1 through A4 stated in Introduction. The endogenously determined asset volatilities and risk premiums evolve stochastically as functions of the latest value of assets. But the volatility process $\{\sigma_t(\tau),\ \tau \in [t, t+1]\}$ and the risk premium process $\{\nu_t(\tau),\ \tau \in [t, t+1]\}$ are still adapted to (conditional on) all the information available ($\mathcal{F}_\tau$), satisfying the requirement for market valuation. We also analyze the nature of dependence of the risk premium-volatility frontiers on the leverage ratio of the firm and on the costs of transaction and monitoring. The extensive form of the game is presented in Figure 1.

The ***market*** is assumed to preclude arbitrage profits by pricing assets via an equivalent martingale measure, Q^ν, with $\nu_t \in (0, \mu_t - r)$ as the risk premium in the market with friction. ***Stockholders*** maximize the net market value of equity of the firm, denoted by S_t, on each date t. Since the firm does not raise any equity on the terminal date T, the net equity value at T as follows:

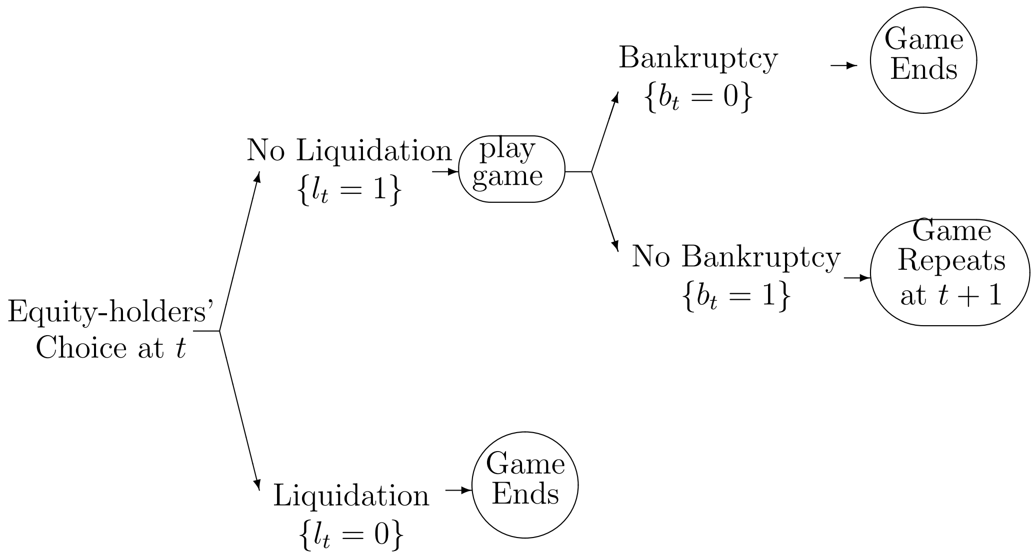

Figure 1
Extensive form of the Game among the equity-holders, debt-holders and the market at time $t, = 0, 1, 2, ...$

$$S_T = (V_T - F)^+ . \tag{4}$$

To preclude arbitrage at $T-1$, the cost of equity-holders' capital, $X_{T-1} + w_{T-1}$, must be equal to the net value created by equity-holders, $E_{T-1}^{Q^\nu}(\beta_{T-1}S_T) - (V_{T-1} - F)^+$, which defines the net equity value at $T-1$,

$$(V_{T-1} - F)^+ = E_{T-1}^{Q^\nu}(\beta_{T-1}S_T) - X_{T-1} - w_{T-1} \equiv S_{T-1}. \tag{5}$$

Proceeding backwards, an ***arbitrage-free net equity value*** atany time t is simply the value of equity on liquidation of the firm at t is as follows:

$$S_t = (V_t - F)^+ = E_t^{Q^\nu}(\beta_t S_{t+1}) - X_t - W_t, \quad t = 0, 1, 2, \ldots, T-1. \tag{6}$$

We consider only arbitrage-free equilibriums in the game. An ***arbitrage-free equilibrium*** in the game under moral hazard consists of seven functions of V_t and F, $\{X_t, \sigma_t, C_t, D_t, b_t, l_t, \nu_t\}$, which solve on each date, $t = 0, 1, 2, ..., T-1$ the following problem.

P1 (Moral Hazard):

$$\begin{array}{cc} \text{Maximize} & (1-b_t)\min(V_t,F) + b_t\left[C_t + E_t^{Q^\nu}\beta_t\left(D_{t+1} - M_{t+1}\right)\right] \\ X_t, \sigma_t, C_t, D_t, \nu_t, b_t, l_t & \\ \text{subject to} & \left[C_t + E_t^{Q^\nu}\beta_t\left(D_{t+1} - M_{t+1}\right)\right] \leq \min(V_t, F), \end{array} \tag{7}$$

$$\begin{array}{ccl} (X_t, \sigma_t, l_t) \in & \text{Argmax} & (1-l)(V_t - F)^+ + \\ & X, \sigma, l \in \{0,1\} & l\left(E_t^{Q^\nu}\left[\beta_t\left(V_{t+1}(V_t + X - C_t, \sigma) - F\right)^+\right] - X - W_t(X)\right) \\ & \text{such that} & E_t^{Q^\nu}\left[\beta_t\left(V_{t+1}(V_t + X - C_t, \sigma) - F\right)^+\right] - X - W_t(X) \\ & & \leq (V_t - F)^+, \end{array} \tag{8}$$

where $V_{t+1} = \tilde{V}_t(t+1)$, $\tilde{V}_t(\tau)$ is generated by the asset value process specified in (2), D_t is the equilibrium market value of debt, ***after*** the debt-holders' monitoring cost is incurred at time t, given by

$$D_t = (1-b_t)\min(V_t, F) + b_t\left[C_t + E_t^{Q^\nu}\beta_t\left(D_{t+1} - M_{t+1}\right)\right], \tag{9}$$

b_t is the equilibrium debt-holder strategy for ***bankruptcy,*** given by

$$b_t = \begin{cases} 0 \text{ (Bankruptcy)} & \Longleftrightarrow \min(V_t, F) > \left[C_t + E_t^{Q^\nu}\beta_t\left(D_{t+1} - M_{t+1}\right)\right], \\ 1 \text{ (No Bankruptcy)} & \text{otherwise,} \end{cases} \tag{10}$$

and l_t is the equilibrium equity-holder strategy for ***liquidation*** of the firm, given by

$$l_t = \begin{cases} 0 \text{ (Liquidation)} & \Longleftrightarrow (V_t - F)^+ > E_t^{Q^\nu}\left[\beta_t\left(V_{t+1} - F\right)^+\right] - X_t - W_t, \\ 1 \text{ (No Liquidation)} & \text{otherwise.} \end{cases} \tag{11}$$

In the moral hazard problem (P1), the equity-holders and debt-holders maximize the market value of debt and equity, respectively, subject to no-arbitrage conditions. The firm ceases to exist after any date t if and only if $l_t b_t = 0$. We can suppose that the bankruptcy court honors debt-holders' appeal for bankruptcy ($b_t = 0$) with probability 1 as long as the choice of b_t is determined by solving the above problem. But this supposition is unnecessary, because we show that $b_t = 0$, whenever V_t is in a specific set of states determined in equilibrium. The debt-holders can simply define the debt covenants at time 0 to declare the firm bankrupt whenever the value of assets of the firm falls in this specific set of states. It is important to observe that only those bankruptcies, which are determined by choices of b_t, guarantee that the equilibrium is arbitrage-free. Observe also that if the equilibrium value of C_t is negative, debt-holders are interpreted to infuse new funds, voluntarily, with no effect on the debt obligations F. The firm operates until $t+1$ only if it is not bankrupt and some investors in the market desire to own the firm at t, i.e., if $l_t b_t = 1$.

To avoid excessive notations, we have assumed that the following supposition holds in equilibrium, which we later show to be true.

Supposition 1: $X_t \geq -(V_t - F)^+$.

For positive values of X_t, Supposition 1 will hold trivially. When X_t is negative, it represents dividend payout. Supposition 1 states that whenever the firm pays a dividend ($X_t < 0$), the amount of payout ($-X_t$) is less than the liquidation value of equity, $(V_t - F)^+$, which is observed by debt-holders. Supposition 1 results in the following lemma shows that equity-holders in the model issue no new capital whenever the firm is liquidated or becomes bankrupt, consistent with the real world.

Lemma 2: Given Supposition 1, $X_t = 0$ whenever the firm is closed $[l_t b_t = 0]$.

First-best (efficient) and Second-best outcomes in Arbitrage-free Equilibrium

Before solving the moral hazard problem (P1), we discuss how the capital market plays the role of a super monitor in our principal-agent problem, where the debt-holders are the principal and the equity-holders the agent.[15] The government passively and efficiently facilitates debt-holders in our model enforce bond covenants via their bankruptcy and coupon requirement strategy (b_t, C_t), and offers secure custody of riskless assets, if chosen by some firms. The government's passive (non-interventionist) role will be clear within the equilibrium in the game is derived.

Conditional on the firm remaining in operation ($b_t l_t = 1$), they "instruct" equity-holders for capital issuance and volatility choices (X_t, σ_t) that are compatible with the equity-holders' incentive of maximization of the market value of their equity, $E_t^{Q^\nu}\left[\beta_t\left(V_{t+1} - F\right)^+\right] - X - W_t(X)$ in (8). An equilibrium without this incentive compatibility requirement, which can be enforced when asset volatility (σ_t) can be costlessly observed is called the "first-best." The principal-agent literature (see, e.g., Harris and Raviv (1979) and Holmstrom (1979)) addresses the issue of obtaining a second-best solution by using the incentive compatibility constraint, i.e., when the agent's control is *ex ante* unobservable.

The most appropriate interpretation of our model is that the market acts as a super monitor to enforce preclusion of arbitrage profits via no-arbitrage conditions for equity, implemented through conditions (7) and (8) and the government passively bestows such power to the market. These conditions are not used in a standard principal-agent problem. These conditions fundamentally distinguish our model from the extant models of agency theory.[16]

In an arbitrage-free equilibrium, the market value of equity S_t is equal to the true value of equity [the maximand in (7)], evaluated at the mutually optimal equity-holder strategy

[15]To focus on the market's super monitor role, we have abstracted away from the equity-holder-manager conflict and the potential role of the managerial labor market in resolving such conflicts.

[16]Observe, however, that Fama's (1980) idea for a competitive managerial labor market can perhaps be viewed as an arbitrage-free pricing of managerial marginal product of labor. It seems that this idea can be modeled to show that the first-best and the second-best will coincide even in a problem where the principal (equity-holders) does not observe the manager's efforts, which affects the payoff of both the principal and the manager. This will involve establishing an arbitrage-free market for the managerial marginal product of labor.

(σ_t, X_t, l_t), the debt-holder strategy (C_t, b_t), and the market value of debt D_t and the asset risk premium ν_t. If such an equilibrium exists,

$$S_t = (1-l_t)(V_t - F)^+ + l_t \left(E_t^{Q^\nu} \left\{ \beta_t \left[V_{t+1}\left(V_t + X_t - C_t, \sigma_t\right) - F\right]^+ \right\} - X_t - W_t(X_t) \right). \quad (12)$$

If an arbitrage-free equilibrium exists, it will ensure a dynamic informational consistency within our model (see, e.g., Riley (1979)). We may not, however, have adverse selection in the sense that, after monitoring on each date t, the agent has no more private information (that cannot change as a result of the game) than that possessed by the principal. Ours is a model of moral hazard in the usual sense since the equity-holders can potentially choose any feasible "effort" $\{\sigma_t(\tau)\}$. Although the agent incurs no cost at the time of choosing a level of effort, current choices of effort impact the *ex post* monitoring cost. Thus, effort choices affect the expected *ex post* monitoring cost and hence the current decision of equity-holders. Ours is also a model of hidden information (Arrow (1984)) in which equity-holders choose their strategy as a function of their observed signals. The signals (sources of hidden information) are the drift $\{\mu_t(\tau)\}$ and the asset value $\{V_t\}$. Given that the firm is not closed at time t, equity-holders' strategy is a map from these observed signals into an effort $\sigma_t(\tau)$ for $t \leq \tau \leq t+1$ as well as X_t. Both the equity-holders and the debt-holders observe the signals symmetrically and imperfectly, although the principal incurs a cost for observing the signals.

The rules for sharing between the principal and agent in our model are determined by the super monitor as the risk premium on equity of equity-holders (agent) and risk premium on debt of debt-holders (principal) if an arbitrage-free equilibrium exists. Equity-holders and debt-holders will then receive *compensations* in the form of risk premiums, which are consistent with their systematic risk of investment in the firm. These systematic risk-expected rate of return relationships are a consequence of the no-arbitrage pricing relationships, and are consistent with the expected instantaneous asset return, $\nu_t(\tau) + r(\tau)$, at time τ, which is determined in equilibrium. These sharing rules may appear unconventional, but they are strikingly similar to standard managerial wage contracts. For example, by a linear wage contract (see, e.g., Holmstrom and Milgrom (1987) and Acharya (1992)), a manager's expected wage is equal to a sum of his wage from an alternative employment, the cost of implementing efforts and a risk-premium.

To show that the first-best and the second-best coincide in an arbitrage-free economy, we consider the following problem under no moral hazard:

P2 (No Moral Hazard):

$$\underset{X_t,\sigma_t,C_t,D_t,\nu_t,b_t,l_t}{\text{Maximize}} \begin{bmatrix} (1-b_t)\min(V_t,F)+b_t\left[C_t+E_t^{Q^\nu}\beta_t\left(D_{t+1}-M_{t+1}\right)\right] \\ \\ (1-l_t)(V_t-F)^+ + \\ l_t\left(E_t^{Q^\nu}\left[\beta_t\left(V_{t+1}(V_t+X_t-C_t,\sigma_t)-F\right)^+\right]-X_t-W_t(X_t)\right) \end{bmatrix}$$

$$\text{subject to} \quad \left[C_t+E_t^{Q^\nu}\beta_t\left(D_{t+1}-M_{t+1}\right)\right] \le \min(V_t,F),$$
$$E_t^{Q^\nu}\left[\beta_t\left(V_{t+1}(V_t+X_t-C_t,\sigma_t)-F\right)^+\right]-X_t-W_t(X_t) \le (V_t-F)^+, \tag{13}$$

where debt-holders' and equity-holders' objectives are simultaneously maximized, subject to the no-arbitrage conditions. Again the bankruptcy (b_t) and the liquidation (l_t) strategies are determined by (10)-(11) and the amount of debt (D_t) is obtained via solution to (9).

In addition to the above specifications of the model, we make the following assumptions:

Assumption 2: $M_t(V_t,F)=F\alpha(v_t)$, where $\alpha(\cdot)\in(0,\bar{\alpha})$, $\bar{\alpha}<1$, is a continuous, convex, and non-increasing function, with $0>\alpha'(v_t)\ge -\frac{1}{\beta_t}+1_{\{v_t<1\}}$ and $E_t^{Q^\nu}\left[\alpha(v_{t+1}(v_t+x-c,\overline{\sigma})\right]=\bar{\alpha}$, for all $\nu\in(0,\mu_t-r)$, where v_t is the asset-to-debt ratio, defined by

$$v_t \equiv \left(\frac{V_t}{F}\right), \tag{14}$$

and $1_{\{\epsilon<1\}}=1$ if and only if $\epsilon<1$ and 0 otherwise, and $\alpha'(v_t)$ is the first derivative of $\alpha(\cdot)$.

Assumption 3: $W_t\equiv w(v_t)(X_t)^+$, where $w(\cdot)$ is a non-increasing convex function.

A first-degree homogeneous monitoring cost, $M(V_t,F)$, in Assumption 2 allows us to focus on the asset-to-debt ratio, v_t by abstracting away from the size of debt F, and makes the analysis tractable. It also makes the interpretation of the monitoring cost per dollar of debt, $\alpha(\cdot)$, more interesting. Monitoring efforts do not increase as the firm becomes more stable, where stability (an indicator of solvency) is defined by the firm's asset-to-debt ratio, v_t. Since V_t is unobservable before monitoring, we need to describe how $\alpha(v_t)$ is determined. Suppose that debt-holders first expend some monitoring efforts, and, if they find it difficult to assess the true value, they examine the firm more closely using additional efforts. The difficulty of assessment is supposed to decrease with the stability of the firm. This process is supposed to continue until the true value of assets as well as the drift is fully determined. Observe that the total monitoring effort, $F\alpha(v_t)$, increases with the level of debt, given v_t. Convexity of $\alpha(\cdot)$ ensures that monitoring efforts decrease at a non-decreasing rate with v_t. The assumption that α does not decrease at too fast a rate over the insolvency region $[\alpha'(v_t)\ge -1/\beta_t+1]$ when $v_t<1$ seems reasonable; we need this to ensure that the market value of debt is not decreasing in the amount of coupon paid by the firm. A sufficient condition for our results is that the total expected monitoring cost reaches an upper bound $\bar{M}=F\bar{\alpha}$, when σ_t reaches its upper bound $\overline{\sigma}$. In fact, given that $v_t\ge 0$, and that $\alpha(\cdot)$ is decreasing, the upper bound

$\bar{\alpha}$ is equal to $\alpha(0)$. We assume that different values of $\nu \in (0, \mu_t - r)$ have no impact on this upper bound; a sufficient condition for this to happen is if $E^{Q^0}[\alpha(v_{t+1}(v_t + x - c, \overline{\sigma}))] = \bar{\alpha}$, which is automatically satisfied for a sufficiently large $\overline{\sigma}$.

Although the specified cost function reflects the nature of monitoring efforts in a real world, our analysis goes through as long as the cost is non-increasing in v_t. For example, the monitoring cost per dollar of debt, $\alpha(\cdot)$, can be specified as $\alpha_0 + (1 - v_t)^+\alpha_1$, where α_0 and α_1 are constants. This cost function implies that the cost of monitoring is equal to $F\alpha_0$ for a solvent firm, and $F\alpha_0 + (F - V_t)\alpha_1$ for an insolvent firm. This cost function can be used to illustrate the existence and properties of our equilibrium. For example, when $M_t = F\alpha_0 + (F - V_t)^+\alpha_1$, the equilibrium can be shown to impose interesting restrictions on α_0 and α_1. If the monitoring cost were zero on all dates, a solution for σ_t would not exist.

Assumption 3 means that the transaction cost per dollar of equity raised (when $X_t > 0$) decreases, or, at least, does not increase with the stability of the firm. This appears to be consistent with the real market place where smaller, less stable corporations seem to pay higher underwriter commissions. More stable corporations generally issue new capital directly, incurring lower transaction costs.[17] Our analysis, however, does not need that $w(v_t)$ be strictly convex and decreasing, as can be illustrated by an example with a constant w. A nontrivial transaction cost is necessary for the firm to choose capital by trading off the cost of raising capital with a possibility of lowering the monitoring cost due to higher capital.

3. Existence of Arbitrage-Free Equilibriums

In either the problem with moral hazard (P1) or without moral hazard (P2), the debt-holders' decision on bankruptcy (b_t) is determined by (10) and the equity-holders' choice for liquidation (l_t) is obtained by (11) if there exists an equilibrium defined by capital issuance, volatility, debt, coupon and risk premium $(X_t, \sigma_t, D_t, C_t, \nu_t)$.

To apply the model to any operating firm, with no regard to when it started, we show the existence of a stationary debt value functional $\hat{D}(V_t, F)$, which is bounded and unique within the class of functionals that are linearly homogeneous in debt F.[18] We can now define the equilibrium market price of debt as:

$$\hat{d}(v_t) \equiv \frac{1}{F}\hat{D}\left(\frac{V_t}{F}, 1\right). \tag{15}$$

Using Assumptions 2 and 3 and dividing both sides of the inequalities and the objective functions [(7)-(8) or (13)], it is now easy to see that if (X_t, C_t) exist in equilibrium, they are first-degree homogeneous in F and stationary functions, denoted by $(\hat{X}(V_t, F), \hat{C}(V_t, F))$

[17] In a sample of 3255 seasoned equity offerings over 1977-1987, the underwriter's fees per dollar of capital raised increased with the ratio of the total liabilities (excluding net worth) to assets of the issuing firm. The t-statistic was 1.5 in a linear regression model. [Data source: US Securities and Exchange Commission Registration Offerings Statistics.]

[18] There may, however, exist non-stationary equilibriums in which the debt value functional may or may not be homogeneous of degree 1 in debt. We show the existence of a homogeneous (of degree 1 in debt), stationary functional primarily for analytical tractability.

. We can thus define the capital infusion and coupon on debt as simpler functions of one variable, the asset-to-debt ratio as follows:

$$\hat{x}(v_t) \equiv \frac{1}{F}\hat{X}\left(\frac{V_t}{F}, 1\right), \quad \hat{c}(v_t) \equiv \frac{1}{F}\hat{C}\left(\frac{V_t}{F}, 1\right), \tag{16}$$

where $\hat{x}(v_t)$ and $\hat{c}(v_t)$ denote, respectively, the amount of equity issued and the coupon rate per dollar of debt of the firm. We shall show that in equilibrium, the volatility, risk premium, bankruptcy and liquidation $(\sigma_t, \nu_t, b_t, l_t)$ are also stationary and first-degree homogeneous in F, and we can thus denote the equilibrium outcomes as simple functions of asset-to-debt ratio of the firm by $(\hat{\sigma}(v_t), \hat{\nu}(v_t), \hat{b}(v_t), \hat{l}(v_t))$. We can also show the existence of an arbitrage-free equilibrium denoted by seven functions of the asset-to-debt ratio of the firm, $(\hat{x}(v_t), \hat{c}(v_t), \hat{d}(v_t), \hat{\sigma}(v_t), \hat{\nu}(v_t), \hat{b}(v_t), \hat{l}(v_t))$, where $\{\hat{\sigma}(v_t)(u), \hat{\nu}(v_t)(u), u \in [t, t+1]\}$ are stochastic processes that depend on v_t and instantaneous time u during a choice period $(t, t+1)$. Thus, the market price of debt in equilibrium, which can be shown to exist in a dynamic programming algorithm, is as follows:

$$\hat{d}(v_t) = (1 - \hat{b}(v_t))\min(v_t, 1) + \hat{b}(v_t)g, \tag{17}$$

where:

$$g \equiv \hat{c}(v_t) + E_t^{Q^\nu}\beta_t\left[\hat{d}(v_{t+1}) - \alpha(v_{t+1})\right], \tag{18}$$

$$v_{t+1} \equiv v_{t+1}(v_t + x - c, \sigma). \tag{19}$$

Note that v_{t+1} is the liquidation value of assets per dollar of debt at time $t+1$ under the original probability measure P and g is the value of debt on non-bankruptcy of the firm. The market value of debt on the left side of (17) is equal to the right side of (17) which represents the higher of the debt values on bankruptcy and non-bankruptcy of the firm. It can be shown that debt-holders will not seek a bankruptcy of the firm *only if* the market price of debt from non-bankruptcy is greater than that from bankruptcy. We can then obtain the following results.

Proposition 2: In either the problem with moral hazard (P1) or without moral hazard (P2), the market price of debt satisfies

$$\hat{d}(v_t) = \min(v_t, 1), \tag{20}$$

and the bankruptcy rule satisfies

$$b = 1 \iff g = \min(v_t, 1) \iff x \geq -(v_t - 1)^+ + \beta_t\alpha(0). \tag{21}$$

Practical Implications of Proposition 2: This proposition shows that in an arbitrage-free equilibrium, the new equity raised must exceed the difference between the maximum discounted monitoring cost $[\beta_t \alpha(0)]$ and the value of equity $(v_t - 1)^+$, conditional on liquidation, which verifies Supposition 1. If an equilibrium exists, trading in debt will generate no arbitrage opportunities, but yield an expected rate of return that is consistent with the risk of debt. In the language of a standard principal-agent model, the principal (debt-holders) receives a fixed payoff equal to the market value of debt, $\min(v_t, 1)$. If this cannot be guaranteed, debt-holders will appeal for bankruptcy by choosing $b = 0$. More importantly, Proposition 2 established efficiency due to arbitrage by rendering both the problems with moral hazard (P1) and without moral hazard (P2) as the same problem (P3) presented as follows:

P3 (Moral Hazard or No Moral Hazard):

$$\begin{array}{ll} \underset{\{x,\sigma,\nu,c,l\}}{\text{Maximize}} & (1-l)(v_t-1)^+ + lf, \\ \text{subject to} & f \le (v_t - 1)^+, \\ & g = \min(v_t, 1), \end{array} \tag{22}$$

where

$$f \equiv E_t^{Q^\nu}\left[\beta_t(v_{t+1}(v_t + x - c, \sigma) - 1)^+\right] - x - w(v_t)(x)^+ \tag{23}$$

is the net equity value per dollar of debt on non-liquidation of the firm and on (v_t, x, c, σ, ν). The market value of equity being maximized in (22) is the equity value on liquidation or non-liquidation, whichever is higher. The equity-holders will obviously seek liquidation only if their value on liquidation is higher than that on non-liquidation of the firm. P3 maximizes the market value of equity subject to the conditions on debt and equity values. The first condition states that the equity value on non-liquidation (f) can be at most equal to the equity value on liquidation, $(v_t - 1)^+$. The second condition makes the market value of debt on bankruptcy, $\min(v_t, 1)$, equal to that on non-bankruptcy of the firm. The set of asset-to-debt ratios for which there exists no solution to P3 defines the closure of the firm or $bl = 0$.

In a standard principal-agent model, the principal's objective is maximized, subject to the constraint that the equity-holders' (agent's) choices are incentive compatible. In our model, the market precludes arbitrage profit opportunities in equilibrium and thus the agent (equity-holders) simply compensates the principal (debt-holders) a market value of debt, $\min(v_t, 1)$, that guarantees a rate of return consistent with the risk faced by debt-holders. If an equilibrium solution exists in our model, it is thus first-best.

If the principal were to generate a higher equilibrium value of debt than $\min(v_t, 1)$, which could be possible without the no-arbitrage condition, then we would not obtain the first-best solution. In addition to an objective of generating as high a value of debt as possible, if the principal in our model costlessly observed the asset volatility (σ), we would have very disturbing results, similar to Mirrlees (1974). This is because the principal would set as high a coupon rate as possible and ask the equity-holders to choose as low an asset volatility as possible, making the equity worthless and the firm non-viable. All these unrealistic possibilities are absent when the market determines prices of securities (asset, debt and equity) by precluding arbitrage profit opportunities in equilibrium.

The following lemma shows the existence of an amount of coupon $\tilde{c} \equiv \tilde{c}(v_t + x, \sigma, \nu)$, with useful properties, that solves the equality of non-liquidation debt value (g) to the liquidation debt value, $\min(v_t, 1)$.

Lemma 3: Suppose that $g = \min(v_t, 1)$. Let the solution (c) to this equation be denoted by $\tilde{c}$. Then (i) $\tilde{c}$ is non-increasing and convex in x, given (v_t, σ, ν). (ii) $\tilde{c}$ is non-decreasing in σ, given (v_t, x, ν). (iii) $\tilde{c}$ is non-decreasing in ν, given (v_t, x, σ).

By supposing that a solution $\tilde{c}$ as in Lemma 3 exists, we can simplify the generalized problem that subsumes moral hazard (P3) as follows.

P4:

$$\begin{array}{ll} \text{Maximize} & (1-l)(v_t - 1)^+ + l f(v_t, x, \tilde{c}, \sigma, \nu), \\ \{x, \sigma, \nu, l\} & \\ \text{subject to} & f(v_t, x, \tilde{c}, \sigma, \nu) \le (v_t - 1)^+. \end{array} \tag{24}$$

The following lemma shows that f has a more useful form if $\tilde{c}$ exists.

Lemma 4: (i) If $\tilde{c}$ in Lemma 3 exists,

$$f(v_t, x, \tilde{c}, \sigma, \nu) = (v_t + x - \tilde{c})\,[R_t - 1] - E_t^{Q^\nu}\,[\beta_t \alpha(v_{t+1})] - w(v_t)(x)^+ + (v_t - 1)^+. \tag{25}$$

(ii) $f_c < 0,\ f_\sigma < 0,\ f_\nu < 0$, where f_j is the partial derivative of $f(v_t, x, c, \sigma, \nu)$ with respect to $j = c, \sigma, \nu$.
(iii) Problem P4 is reduced to:

P5:

$$\begin{array}{lll} \text{Given } (v_t, \nu), \text{ solve} & f(v_t, x, \tilde{c}, \sigma, \nu) = (v_t - 1)^+ & \text{for } \sigma \\ \text{such that} & x \in \arg\max_{x'} f(v_t, x', \tilde{c}(v_t + x', \sigma, \nu), \sigma, \nu). & \end{array} \tag{26}$$

The objective function (f) in (25) represents the net value of equity, conditional on no liquidation. It is equal to the net value of assets carried forward $(v_t + x - \tilde{c})$ times the expected return on equity per dollar invested in net assets $(R_t - 1)$ minus the sum of the expected discounted costs of *ex post* monitoring by debt-holders and of raising any new equity plus the current liquidation value of equity $(v_t - 1)^+$.

In an arbitrage-free equilibrium, the monitoring cost is ultimately borne by the firm. Further, since the net equity value in equilibrium is equal to $(v_t - 1)^+$, as in (26), equity-holders' choices must result in an equality of the incremental rise in the market value of equity, $(v_t + x - \tilde{c})(R_t - 1)$, with the expected discounted costs due to monitoring and capital issuance $E_t^{Q^\nu}\,[\beta_t \alpha(v_{t+1})] + w(v_t)(x)^+$. This means that the market acting as a super monitor compensates the debt-holders and equity-holders an incremental market value equal to the minimum possible discounted expected monitoring and equity issuance costs. The incremental market value is achieved via the equivalent state pricing measure.

In the following proposition, we show that there exists a unique no-arbitrage equilibrium relationship between the expected rate of return to assets and the asset volatility of a firm, given the current asset-to-debt ratio (stability) of the firm (v_t),

Proposition 3 (Efficiency): Suppose that Assumptions 1 through 3 hold.
(i) There exists a unique value of new capital (x) that maximizes the value of equity conditional on continuance of the firm $f(v_t, x, \tilde{c}, \sigma, \nu)$ and on asset value, volatility and risk premium (v_t, σ, ν). Let the maximizing x be denoted by $\tilde{x}(v_t, \sigma, \nu)$.
(ii) The continuing firm's equity value $f(v_t, x, \tilde{c}, \sigma, \nu)$ is monotonically decreasing in volatility (σ), given the asset value, new capital and risk premium (v_t, x, ν).
(iii) The equity value $f(v_t, x, \tilde{c}, \sigma, \nu)$ is monotonically increasing in risk premium (ν), given asset value, new capital and volatility (v_t, x, σ).
(iv) The problem P5 is reduced to a must simpler problem P6 as follows.

P6:

$$\text{Given } (v_t, \nu), \text{ solve } \tilde{f}(v_t, \sigma, \nu) = 0 \qquad \text{for } \sigma, \tag{27}$$

where the reduced function for the net value of equity from continuance minus the value of liquidation of the firm is given by

$$\tilde{f}(v_t, \sigma, \nu) \equiv f(v_t, \tilde{x}(v_t, \sigma, \nu), \tilde{c}(v_t + \tilde{x}(v_t, \sigma, \nu), \sigma, \nu), \sigma, \nu) - (v_t - 1)^+. \tag{28}$$

the net value of equity $\tilde{f}(v_t, \sigma, \nu)$ is monotonically
(v) decreasing in volatility (σ), given asset value and risk premium (v_t, ν).
(vi) decreasing in risk premium (ν), given asset value and volatility (v_t, σ).
(vii) increasing in asset value (v_t), given volatility and risk premium (σ, ν).
(viii) Define for every $\nu \in (0, \mu_t - r)$ the lowest asset value (per unit of debt) for which the firm will remain open ($\tilde{f} \geq 0$):

$$\hat{v}(\nu) \equiv \inf_{v \geq 0} v \qquad \text{such that} \qquad \tilde{f}(v_t, \sigma, \nu) \geq 0. \tag{29}$$

The firm will come into existence only if $\hat{v}(0) < \infty$. Further, if $\hat{v}(0) < \infty$, there exists a solution to P6, denoted by $\tilde{\sigma}(v_t, \nu)$ for every feasible $\nu \in (0, \mu_t - r)$ and $v_t > \hat{v}(\nu)$. The firm is closed [$bl = 0$] if and only if the current value of asset (per unit debt) is less than a minimum threshold ($v_t < \hat{v}(\nu)$). The minimum threshold asset-to-debt ratio above which a firm remains open is a function of the risk premium.
(ix) If the firm remains in operation ($v_t > \hat{v}(\nu)$), there exists a volatility function $\tilde{\sigma}(v_t, \nu)$ which solves (27) to generate a zero incremental equity value from continuance as compared to liquidation value:

$$\tilde{f}(v_t, \tilde{\sigma}(v_t, \nu), \nu) = 0,$$

which is solved for a risk premium $\hat{\nu}(v_t)$, given the asset value per unit of debt, v_t. Then the risk premium ($\hat{\nu}(v_t)$) and the corresponding volatility ($\hat{\sigma}(v_t)$) obtain as functions of the current asset value v_t such that

$$\tilde{f}(v_t, \tilde{\sigma}(v_t, \hat{\nu}(v_t)), \hat{\nu}(v_t)) = 0,$$

and $\hat{\sigma}(v_t) \equiv \tilde{\sigma}(v_t, \hat{\nu}(v_t))$, $\hat{x}(v_t) \equiv \tilde{x}(v_t, \hat{\sigma}(v_t), \hat{\nu}(v_t))$ and $\hat{c}(v_t) \equiv \tilde{c}(v_t + \hat{x}(v_t), \hat{\sigma}(v_t), \hat{\nu}(v_t))$.

Note $\hat{v}(\nu)$ is the lowest asset-to-debt ratio above which the continuing equity value (f) is larger than the liquidation value $(v_t - 1)^+$, when $\tilde{f}(v_t, \sigma, \nu) = f - (v_t - 1)^+ \geq 0$ holds.

Practical Implications of Proposition 3: Proposition 3 defines a frontier (graph) of the risk premium versus volatility for every asset-to-debt ratio greater than a minimum threshold ($v_t > \hat{v}(\nu)$) over which the firm remains open [$bl = 1$]. The frontier relates the instantaneous expected asset return $\hat{\mu}_t(\tau) = \hat{\nu}_t(\tau) + r(\tau)$ to the volatility $\hat{\sigma}_t(\tau),\ \tau \in [t, t+1]$ of the firm. The firm is indifferent among all pairs of $\{\hat{\nu}(v_t), \tilde{\sigma}(v_t, \hat{\nu}(v_t))\}$ on this frontier. In Acharya (1990), a similar problem is analyzed for banks with a fixed risk premium ν, resulting in a unique solution, given v_t, which includes a unique volatility and a unique threshold asset-to-debt ratio below which a bank is optimally closed.

The equilibrium expected return-volatility frontier we obtain here seems very interesting, because it describes a cross-section of firms (including banks) whose expected return on assets depend on the volatility they choose. This result appears to be an economically significant departure from the standard paradigm in which the expected return to assets and volatility are presumed to be unrelated. Our model with no friction and no moral hazard reduces to the standard paradigm (Capital Asset Pricing Model or the Arbitrage Pricing Theory) with a cross-sectional relationship between expected returns and systematic risks (betas). This frictionless model does not, however, imply any relationship between expected return, μ_t, and volatility, σ_t. Indeed σ_t is indeterminate if there is moral hazard and no friction as in Chan, Greenbaum and Thakor (1991).

A very interesting endogenous relationship between the expected return ($\hat{\mu}_t(u) = \hat{\nu}_t(u) + r(u)$) and the volatility choice of a firm ($\hat{\sigma}_t(u)$) obtains in a arbitrage-free market with friction and moral hazard in the game between debt-holders and equity-holders.

The practical implications of Proposition 3 are profound: The algorithm for the moral hazard problem - faced by debt-holders due to equity-holders' control of the *ex ante* unobservable asset volatility - is the same as that for the absence of moral hazard. This robust result obtains because our model incorporates many *real world* factors like arbitrage pricing and trading, monitoring by debt-holders, new capital issuance by equity-holders, and gaming between equity-holders and debt-holders with the market behaving like a super monitor. Negative new capital is dividend payout. The existing models on moral hazard do not incorporate such real world factors and so obtain inefficiency or second best outcomes due to moral hazard. If arbitrage trading is not allowed within our model, we would also obtain inefficient outcomes. Proposition 3 demonstrates that the moral hazard problem is efficiently resolved due to arbitrage trading and pricing in the market place.

The existing belief that inefficiency is natural due to moral hazard has induced government intervene, for example, in the banking sector. Government intervention imposes inefficiency. The current presumption is that the inefficiency of government intervention is less costly than moral hazard. Proposition 3 makes such presumption unwarranted by establishing that the market can act as a super monitor to achieve efficiency. Proposition 3 thus obviates calls for government intervention to supplant the efficient act of the market.

Most of the existing research in economics, except that on moral hazard, *assumes* that the market is supreme, though not in a more general constitutionally specified market economy, characterized by axioms A1 through A4 stated in Introduction. Proposition 3 *proves* that this market is indeed supreme in this generalized constitutional economy. This economic environment shows that arbitrage trading leads to efficiency and first best outcomes that cannot be achieved otherwise. Proposition 3 thus proves as true the common belief in the developed world that the market in a constitutional economy should be supreme to achieve efficient outcomes. This result cannot be dragged to vouch for supremeness of the market in an economy that is not constitutional.

Inefficient outcomes in the models of moral hazard has been the basis for government intervention because, as the current argument goes, such intervention imposes a lower tax on society than the cost of inefficiency due to moral hazard. Proposition 3 obviates such arguments.[19]

Proposition 4 examines the nature of relationship between asset volatility and risk premium and how this relationship shifts with respect to the asset-to-debt ratio of a firm (v_t). This proposition also derives the nature of the threshold asset-to-debt ratio ($\hat{v}(\nu)$) above which firms operate in arbitrage-free market equilibrium.

Proposition 4 (Asset Volatility - Risk Premium Frontier):
(i) Given the asset value per dollar of debt (v_t), the asset volatility ($\tilde{\sigma}(v_t, \nu)$) is a monotonically decreasing function of asset risk premium (ν).
(ii) Given risk premium (ν), the asset volatility $\tilde{\sigma}(v_t, \nu)$ is a monotonically increasing function of the asset value v_t.
(iii) There exists an equilibrium risk premium $\tilde{\nu}(v_t, \sigma)$ as a function of volatility (σ) and asset value (v_t). The risk premium is a monotonically decreasing function of asset volatility (σ), given asset value (v_t).
(iv) The risk premium is a monotonically increasing function of asset value (v_t), given asset volatility (σ).
(v) The minimum threshold asset value per dollar of debt $\hat{v}(\nu)$ is a monotonically increasing function of the risk premium.

[19] An indirect implication not modeled in this paper is that sovereign countries should allow their currencies to trade freely to achieve first-best (efficient) outcomes for the global society. Smaller developing economies may, however, find it optimal to determine exchange rates by balancing the costs of banking and social instability with the benefits of job creation, as argued in Acharya (2007).

Practical Implications of Proposition 4: This proposition presents an interesting testable inverse relationship between asset return and volatility. Our model does not explicitly establish relationships between volatility and return of portfolios or common stocks. Yet, our asset volatility-return relationship can be exploited to derive a similar negative relationship between common stock volatility and return by making adjustments for volatility of debt.

The second interesting implication of Proposition 4 is that as firms become more stable, with the stability indicated by v_t, they are able to take greater asset volatility risk defined by σ. The intuition for this result is that more stable firms expect to incur lower monitoring costs, given volatility. The expected return would increase if the expected monitoring cost decreased due to a rise in the asset-to-debt ratio, unless the volatility increased sufficiently to increase the expected monitoring cost and keep the expected return unchanged. This means that, given the expected return, the volatility increases with the asset-to-debt ratio of a firm. Further, as Proposition 5-(iv) shows, the expected return to assets will increase with the asset-to-debt ratio of a firm, given volatility. These implications also offer very interesting and novel tests of our theory. Casual empiricism shows that more stable firms experience greater momentum in their prices. It is because they can afford to take greater risks.

Proposition 3 shows that the firm will be closed in equilibrium if its asset-to-debt ratio (v_t) falls below a minimum threshold $\hat{v}(\nu)$, which is a function of the asset risk premium. Proposition 4-(v) shows that this threshold is an increasing function of the asset risk premium. This is because the market uses the risk premium to discount the future payoff to the asset. A higher risk premium lowers the current value of the future payoff. This prompts the debt-holders to set a higher minimum asset-to-debt threshold for higher risk premium on assets. This result contrasts the fixed thresholds of asset-to-debt ratio for bank closure derived in Acharya and Dreyfus (1989) and Acharya (1990), where ν is assumed to be given (cf. Diamond (1984) and Smith and Warner (1979)). The threshold $\hat{v}(\nu)$ we derive here defines the equilibrium bond covenants that debt-holders would impose on equity-holders in an arbitrage-free equilibrium.

As a practical matter $\hat{\mu}_t(u) = \hat{\nu}_t(u) + r(u)$ could be estimated from data as the cost of capital of assets. The threshold asset-to-debt ratio that debt-holders could design for seeking bankruptcy of the firm could be set as a decreasing function of this estimated cost of capital. This seems to have a very useful implication for bank regulatory policies. Suppose that a bank regulator chooses to act as a surrogate for bank debt-holders. Then the regulator will mimic our model's equilibrium debt covenants obtained for a deregulated industry operating in an arbitrage-free market economy. The covenants are (i) minimum threshold asset-to-debt ratio below which to close banks and (ii) debt/deposit insurance premium. The regulator will insure bank debt and deposits for a fair price equal to the equilibrium coupon on debt in our model.[20] The minimum threshold asset-to-debt ratio below which banks should be closed

[20]Black, Miller and Posner (1978) lucidly discuss the similarity between the lender-borrower relation and the regulator-bank relation. Although, they do not analyze a model with moral hazard, they argue that both the borrower and the lender have an interest in minimizing ‘‘administrative costs of lending." Administrative costs within our model would be the expected monitoring and equity issuance cots.

will be an increasing function of the bank asset risk premium, as implied by Proposition 4. The equilibrium bank regulatory policy, which would prevail in an arbitrage-free economy with deregulated banks, is the same as the one that the surrogate regulator would find optimal in the context of our model.[21] Observe that the risk premium is an increasing function of systematic risk if we impose the cross-sectional Capital Asset Pricing Model on our results. Then a higher bank systematic risk will imply a greater minimum threshold asset-to-debt ratio below which the bank should be closed, if the regulatory policy is to mimic an unregulated banking industry in an arbitrage-free economy.

The result that the minimum threshold asset-to-debt ratio is monotonically increasing with the volatility risk of a firm offers another novel test of our theory. Everything else equal, operating firms with higher volatility risk have higher asset-to-debt ratio (lower leverage) than those with lower asset-to-debt ratio in an arbitrage-free economy. One can design trading strategies based on deviation of market prices of assets from their fair values implied by the frontier of asset values and volatilities constructed for a cross-section of all publicly trades firms.

4. Constitutional System of Governance

This section shows that in equilibrium the government creates a *Safe Bank* to hold the safe asset (deposits) chosen by some firms and households.

Proposition 5 (Safe Bank): The safest asset with least volatility in the model offers a return equal to the riskless rate of interest or zero risk premium. The safe assets (safe deposits) are held by firms unwilling to take risk. The only player in our model that can secure the safe deposits to offer the riskless interest rate is the government. The government can (a) provide a guarantee for the safe deposits at a risky financial firm by collecting a deposit insurance premium to cover for any potentil loss based on the risk chosen by the financial firm or (b) directly hold the safe deposits in a government entity, called *Safe Bank*. Option (a) is neither efficient nor constitutional. Option (b) is efficient and constitutional. (c) Government insurance of deposits in risky financial firms is unnecessary as a panic-prevention mechanism, and unconstitutional.

Proof of Proposition 5: (a) The deposit insurance premium is a cost to the financial firm that will reduce its expected return to a level below that implied by the efficient frontier of risk and premium developed in Proposition 3. This will make the financial firm inefficient, unless the financial firm restores its efficiency by paying the depositors a lower interest rate equal to the riskless rate minus the deposit insurance cost per dollar. But the cost of deposit insurance passed on to the depositors will make the firms choosing riskless assets earn, inefficiently, less than the riskless rate they ought to received based on the risk-return efficient frontier as per Proposition 3. This inefficiency is unconstitutional by axiom A4 stated in Introduction.

[21] Acharya (2003) argues that *safe banking* will obviate bank regulation in the best interest of taxpayers.

(b) The *Safe Bank* does not involve custody of riskless assets in risky financial firms or the cost of deposit insurance and is, therefore, efficient in payment of the riskless rate to firms choosing safe assets (deposits). Efficiency makes the *Safe Bank* constitutional by axiom A4.
(c) The existing literature presumes (i) that safe deposits must be held in risky financial firms and (ii) that depositors may panic due to some external forces of nature like sun spots or invisible hands to justify a government deposit guarantee via holding of riskless deposits in risky financial firms. The *Safe Bank* renders both the presumptions superfluous, making the government deposit insurance via custody of riskless deposits in risky financial firms unnecessary and unconstitutional by proofs of parts (a) and (b).

The inefficiency, and hence unconstitutionality, stems from the avoidable cost of warehousing riskless assets in risky financial firms. This cost can be avoided only if the safe assets are kept separately from the risky financial firms. In our model, only the government can keep the safe assets securely and separately without imputing the risk in a government entity, named here as *Safe Bank*. The maintenance cost of the secure custody of the safe assets in the centralized government entity, *Safe Bank*, is also less than the maintenance cost of custody in a divers group of risky financial firms. This makes the *Safe Bank* even more efficient than the option in Proposition 5(b), though our model does not specify these costs of maintenance. The *Safe Bank* will be identical to the Central Bank or the Federal Reserve of the U.S. if the Federal Reserve Act (FRA) of 1913 is duly amended to conform the amended FRA to the constitutional axiom A4 stated in Introduction. Our model thus proves the existence or necessity of a constitutionally mandated central bank, called the *Safe Bank*.

Corollary to Proposition 5 (Role of Governance for Secure Custody of Safe Assets): The government will provide custody of safe assets only via risky financial firms in equilibrium in our model if it cannot enact and enforce laws as per constitution, i.e., when axioms A2 and A4 are voided.

Proof of Corollary to Proposition 5: The proof follows immediately from the arguments presented in Proposition 5.

The Federal Deposit Insurance Corporation was conceived as a part of the Glass-Steagall Act of 1933 to profide government guarantee of safe deposits in risky financial firms. The government then had little power. The surviving financial firms wielded enormous economic power. They crafted the idea of government guarantee of safe deposits (up to a ceiling) in risky financial firms. This was not only unconstitutional but also inefficient, as was vivid during the Great Recession of 2008" The federal deposit insurance could not prevent the runs in the $3 trillion money markets, and bank deposits above the insurance ceiling. Before 2007, the FDIC collected too little premium to fund the severe insurance losses that resulted after the Great Recession of 2008. The FDIC resoted to mergers of even solvent banks like Washington Mutual with larger financial institutions, perhaps to solidify the larger firms to prevent systemic runs. But it sparked systemic runs as the holders of more than $3 trillion in money markets (a lot of which was invested in risky financial firms like Lehman Brothers that failed in 2008) panicked, bank debt holders lost trust in their banks, and bank depositors with more than $100000 lost sleep. The government landed up extending even more insurance, i.e., of the money market funds, all bank debt and deposits up to $250000, and even new bank debt. It is as if the government is naturally heading towards enactment of the properties of the *Safe Bank*.

The Federal Reserve Act of 1913 offers unequal privilege only to financial firms, and the most privilege to the largest of the financial firms, as presented in detail in Introduction: (i) The Federal Reserve Board (FRB) offers secured custody to safe deposits of financial firms and foreign central banks, not nonfinancial firms and households. (ii) The FRB grants financial firms the privilege of earning a significant interest rate spread on money borrowed from savings of other firms including households and from the FRB at the FRB-dictated lower rate and lend the same to the government at a higher interest rate. (iii) The FRB allows only the largest privileged financial firms to participate in primary Treasury auctions to earn the highest possible rate from lending to the government; this rate is not feasible for the rest of the firms including smaller financial firms and households. The privileged financial firms take no risk due to such borrowing at lower rate and lending at higher rate to the government. Neither do they toil or serve anyone for the incredible guaranteed financial privilege. The Federal Reserve Act allows the privileged financial firms to extract at no risk from those who toil and risk in enterprise, creativity, innovation and service to enhance national prosperity and security. The following proposition establishes that such unequal privilege makes the Federal Reserve Act inefficient and unconstitutional.

Proposition 6: (a) The Federal Reserve Act of 1913 is neither efficient nor constitutional, i.e., not supported in equilibrium of our model. (b) This Act can obtain in equilibrium within our model only if the government cannot enact and enforce constitutional rules of law (when axioms A2 and A4 are voided).

Proof of Proposition 6-(a): The Federal Reserve Act prevents households from earning the highest interest rate possible on their safe deposits, as per the risk-return frontier obtained in Proposition 3, that they could earn by lending the government directly through the *Safe Bank*. By this Act, privileged financial firms earn a higher interest rate on their safe deposits (capital reserve accounts held at the Federal Reserve and funds lent to the government in primary Treasury auctions. The Act is thus inefficient and unconstitutional. **6-(b)** If the government is incapable of enacting and enforcing constitutional rules of law (when axiom A2 and A4 are voided in our model), the privileged financial firms will have the power to legislate the Federal Reserve Act of 1913 to maximize their net-worth according A1 and A3 of our model.

Practical Implication of Proposition 6: This proposition necessitates that the Central Bank of a country ought to be like the *Safe Bank*, characterized by Proposition 6, to provide secure custody of riskless savings of all firms including households for efficient funding of the government. The *Safe Bank* is efficient and constitutional. It makes the government efficient (least costly) and the economy liberated. *Liberating the economy* means all firms (financial, nonfinancial and households) receive equal treatment, guaranteed by ther constitution, and earn rates of return consistent with the risks of their assets, ensured in equilibrium. A liberated economy enshrined in the constitution does not permit any group of firms to legislate laws (by lobbying the Congress) to garner special privilege to earn abnormal rates of return (above the efficient risk-return frontier) by imputing inefficient rates of return (below the risk-return frontier) on assets of other firms.

With *Safe Banking*, all firms including households will preserve and enhance their wealth as per the constitution and earn efficient returns as per the risks of their chosen assets in equilibrium. The currently chartered Central Banks around the world cannot guarantee this, as was vivid during the Great Depression and the Great Recession. Inducing perseverance and not rewarding indolence in society will enhance prosperity, security and stability of the nation. The proposed system will thus reduce the chances of financial depression, and promote general welfare and domestic tranquility as dreamed by the American founding fathers.

Safe Banking will provide the security needed for riskless deposits of all firms including households to avert irrational panics. *Safe Banking* will basically extend the privilege now available to only financial firms to all firms including households.

Within the current system of money and finance in the U.S., the quantum of wealth transfer from the persevering nonfinancial firms including households to the financial firms is staggering, for example, when the rate of interest on deposits is 1.5% and the rate paid by the government is 4% on a total of $6 trillion in bank deposits. The spread is 2.5%. The unconstitutional transfer to privileged financial firms from the rest is a staggering annual sum equal to .025 x 6,000,000,000,000 = 150,000,000,000. In this example, the current monetary system recklessly transfers a staggering quantum of wealth of $150 billion, per year, from the hard earned savings of *We the People* to a few privileged financial firms who continually cause depressions and recessions to jeopardize national tranquility and general welfare. This is unconstitutional. It can and should be stopped immediately by adopting the proposed constitutional monetary system through amendment of the Federal Reserve Act of 1913 and elimination of the FDIC.

In the proposed monetary system, financial firms can no longer act as middlemen to pocket a significant interest spread without risk or toil. The proposed system will preclude the windfall transfer of wealth from *We the People* and from the government of *We the People* to a few financial firms.

The prevailing wealth transfer system amounts to an unconstitutional deprivation of individual property and liberty. This leads to an unsustainable surreptitious (financial) subjugation by a few privileged financial firms of *We the People*, who enterprise, produce, create, innovate and serve to make the nation prosperous and secure.

The model shows that the current system will prevail as long as the government (Congress and President in the U.S.) will not or cannot enact and enforce rules of law as per the constitution. This could happen, for example, when the government was enfeebled and dictated by the largest surviving financial firms in the aftermath of the 1907 banking panics and runs. The powerful bankers that survived the banking panics and runs could craft the Federal Reserve Act of 1913 and persuade the Congress to pass it to create a central bank that would operate at their behest. The charter of this central bank (the Federal Reserve) only aggravated the financial crisis and led the economy to the catastrophic Great Depression.

Safe Banking without federal deposit insurance vs. Dodd-Frank Financial Regulatory Bill of 2010:

A major direct policy implication of the above propositions is that *Safe Banking* without federal deposit insurance and without government intervtion is efficient and constitutional. The Dodd-Frank Financial Regulatory Bill of 2010 preserves inefficiency and unconstitutionality by letting continuance of the federal deposit insurance and government intervention in resolution of financial firms. The Dood-Frank bill is eerily silent about the unconstitutionality and inefficiency of the Federal Reserve Act of 1913 and of the FDIC. A complete discussion on unconstitutionality of the current system of money and finance is addressed in Acharya (2010).

The U.S. government now insures bank deposits to circumvent the systemic risk due to panic-driven bank runs. But the government deposit insurance can lead to moral hazard, which is defined as banks using insured deposits for excessively risky bets. Consider, for example, that a bank starts with 8 dollars of common equity and 92 dollars of insured deposits and then creates a firewalled subsidiary with equity of 100 dollars infused from the parent bank. The subsidiary then borrows 900 dollars of insured deposits. The parent bank thus meets the statutory minimum required capital-to-assets ratio of 8% and its subsidiary has 10%. On a consolidated basis, though, the bank has only 8 dollars of equity for 1000 dollars of assets or 0.8% capital-to-assets ratio and is thus badly under-capitalized. The executives of this bank may be tempted to bet the insured funds in the firewalled subsidiary on very risky investments. Such risky bets can generate high returns to the parent bank's shareholders, though with a small probability, but subject the insured deposits to the risk of losses to be borne by taxpayers.[22] The current banking crisis is primarily confined to under-capitalized banks with risky firewalled subsidiaries, known as conduits or trusts or super investment vehicles.

The government is currently obligated to monitor and regulate insured banks to alleviate moral hazard. The supposition is that bank deposit insurance and regulation are less costly to society than the systemic burden due to panic-driven runs. Such supposition is, however, redundant when the market can achieve first-best efficiency through trading as shown in Proposition 3. The banking moral hazard stems from excessive risk taking, infrequent trading of assets within firewalled subsidiaries, and the illiquidity due to the government's restrictions on auctioning off regulated bank assets. The large insured U.S. banks have perhaps accumulated too much risk in their firewalled subsidiaries due to a tacit government

[22]The top management of a major U.S. bank once was incensed when the author asked for maintenance of the minimum regulatory capital requirement on a consolidated basis.

policy that such banks are too big to fail. In the current predicament, the regulators are reluctant to let banks auction off their risky assets at "throw away" prices because of potential reprisal from taxpayers.

Arbitrage trading induced efficient deregulation leads within our model to a **safe banking policy**, which necessitates the government to amend the Federal Reserve Act of 1913 and eliminate the FDIC (i) to stop government guarantee of riskless deposits at risky financial firms and (ii) to have the Federal Reserve offer secure custody of riskless deposits of all firms including households and eliminate every special privilege now given to the financial firms. The Federal Reserve Board then become the *Safe Bank* of our model.

This *safe banking policy* is the consequence of a constitutional market economy characterized by axioms A1 through A4 stated in Introduction. This policy makes the economy and the government efficient without the adverse costs of government regulation and intervention. The *safe banking policy* will circumvent systemic risks due to banking panics and runs and eliminate the current system of federal deposit insurance to allow markets to effect efficiency in banking and financial markets with little regulation. The *safe banking policy* here achieves the goal in Acharya (2003), which does not address constitutionality and efficiency as done here.

Trading has practically helped the market in the efficient collapse of the largest universal bank (Drexel Burnham Lambert) in the late 1980's, liquidation of the largest hedge fund in the late 1990's, dismantling of several large hedge funds in 2007. But the government did not allow trading of bank assets when the markets collapsed as the entire banking industry tottered in 2008. The government rather used the Federal Reserve to buy the "illiquid" or "toxic" bank assets at prices far greater than what these assets would have fetched if the market participants were allowed to trade freely in 2008. The public severely criticized such bailout of especially big financial firms.

The Dodd-Frank bill promises to prevent such bailouts in future through an orderly liquidation of all banks (including the biggest) whose capitals fall below a certain increased threshold of capital-to-assets ratio. This is a *promise* to not let the too big to fail banks continue when they flounder. This *promise* was incorporated in the bill to soothe the public outrage about the bailout of the largest banks like Citigroup and Goldman Sachs. But this *promise* is impossible to keep when the largest banks are becoming larger, naturally due to the federal deposit insurance administered by the Federal Deposit Insurance Corporation (FDIC) which has the incentive to merge failing financial firms with the ones to save its deposit insurance fund.

After listening to the testimonies before the Financial Crisis Inquiry Commission of the FDIC and other regulatory agencies on September 2, 2010, it is very obvious that the primary *goal* of the FDIC is to minimize the cost to its Deposit Insurance Fund (DIF). So, what is the best policy for the FDIC to achieve its goal of minimizing the cost to the DIF? The answer is to pick one bank and to let this bank acquire the rest of the banks. The DIF will then have no cost as long as the single remaining gigantic bank stays solvent. The FDIC goal will then lead to a recommendation to the Congress to close this gigantic bank while it remains solvent!

The FDIC has led to the creation of biggest banks according to data presented by the FCIC. The FDIC *goal* is thus very unseemly. It simply ignores the antitrust laws, the panic it spreads when stakeholders' investments are wiped out upon seizure of solvent banks like the Washington Mutual, the systemic risk such seizures create, and the unconstitutional usurpation (indeed destruction) of the hard-earned private capital invested in securities of seized solvent banks.

Indeed, the FDIC goal since its existence has led to fewer banks that are growing larger over time with arranged acquisitions of rival banks (big and small). Extrapolating this current reality into the future should show that the U.S. will ultimately have a few gigantic banks after they swallow the rest with the FDIC intervention. The Congress cannot obviously allow such gigantic banks and will rather follow anti-trust laws to break the largest remaining banks and even contemplate closing the FDIC that has pursued for an unsustainable goal leading to a nightmarish predicament.

Given the above potential reality in the future, what is optimal policy for the Congress now? The answer is to close the FDIC now and to provide *Safe Banking* facility to all firms including households without the current federal deposit insurance of deposits in risky financial firms. Closing the FDIC now will obviate the ongoing unconstitutional usurpation of the hard-earned private capital invested in solvent banks being closed by the FDIC to accomplish its unsustainable goal. Closing the FDIC now will also prevent formation of one or a few unwanted gigantic banks with little competition in the industry that violates the existing anti-trust laws.

5. Conclusion

The most significant contribution of this paper is to establish that the market within a constitutional economy can be an efficient super monitor to resolve moral hazard efficiently.

We show the existence of an efficient and constitutional entity called *Safe Bank* created by the government to provide a safe custody of riskless assets (deposits) of all firms, including households, that choose the risless asset.

The *Safe Bank* averts panics in equilibrium, making the federal guarantee of deposits at risky financial firms superfluous. We prove that the FDIC and its federal deposit insurance and the Federal Reserve Act of 1913 are inefficient and unconstitutional.

If the Federal Reserve Act of 1913 is amended to allow the Federal Reserve to give equal privilege to all firms (not just financial firms) and if the FDIC is eliminated the new Federal Reserve will be identical to our *Safe Bank* to make the economy efficient and constitutional.

Equilibrium debt covenants like minimum threshold asset-to-debt ratio for firms to remain in operation are obtained. In equilibrium, (a) the asset risk premium is negatively related to volatility of assets of a firm, (b) the asset volatility and risk premium are increasing functions of the asset-to-debt ratio, and (c) the minimum threshold asset-to-debt ratio below which the firm goes bankrupt is an increasing function of the asset risk premium.

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The Appendix

Proof of Lemma 1: We outline the proof here. For $\tau \in [t, t+1]$,

$$
\begin{aligned}
& \tilde{\beta}_t(\tau)\tilde{V}_t(\tau)\xi_t(\tau) \\
& = (V_t + X_t - C_t)\xi_t(t) \times \\
& Exp\left(\int_t^\tau (\mu_t(u) - r(u))du - \frac{1}{2}\int_t^\tau \left[\sigma_t^2(u) + \left(\frac{\nu_t(u)}{\sigma_t(u)}\right)^2\right] du + \int_t^\tau \left[\sigma_t(u) - \frac{\nu_t(u)}{\sigma_t(u)}\right] dZ(u)\right) \\
= \; & (V_t + X_t - C_t)\xi_t(t) Exp\left(-\frac{1}{2}\int_t^\tau \left[\sigma_t(u) - \frac{\nu_t(u)}{\sigma_t(u)}\right]^2 du + \int_t^\tau \left[\sigma_t(u) - \frac{\nu_t(u)}{\sigma_t(u)}\right] dZ(u)\right) \times \\
& Exp\left(\int_t^\tau (\mu_t(u) - r(u))du - \frac{1}{2}\int_t^\tau \left[\sigma_t^2(u) + \left(\frac{\nu_t(u)}{\sigma_t(u)}\right)^2\right] du + \frac{1}{2}\int_t^\tau \left[\sigma_t(u) - \frac{\nu_t(u)}{\sigma_t(u)}\right]^2 du\right), \\
= \; & (V_t + X_t - C_t)\xi_t(t) Exp\left(-\frac{1}{2}\int_t^\tau \left[\sigma_t(u) - \frac{\nu_t(u)}{\sigma_t(u)}\right]^2 du + \int_t^\tau \left[\sigma_t(u) - \frac{\nu_t(u)}{\sigma_t(u)}\right] dZ(u)\right) \times \\
& Exp\left(\int_t^\tau (\mu_t(u) - r(u) - \nu_t(u))du\right),
\end{aligned}
$$

which is a martingale under P if and only if $\mu_t(u) - r(u) - \nu_t(u)$.

By Girsanov's theorem, the equivalent Brownian motion under Q is

$$
dZ^*(u) = dZ(u) - dZ(u)\left(\frac{d\xi_t(u)}{\xi_t(u)}\right) = dZ(u) - dZ(u)\left(-\frac{\nu_t(u)}{\sigma_t(u)} dZ(u)\right) = dZ(u) + \left(\frac{\nu_t(u)}{\sigma_t(u)}\right) du,
$$

which implies that, under Q,

$$
\begin{aligned}
& \tilde{\beta}_t(\tau)\tilde{V}_t(\tau) \\
& = (V_t + X_t - C_t) Exp\left(\int_t^\tau (\mu_t(u) - r(u))du - \frac{1}{2}\int_t^\tau \sigma_t^2(u)du + \int_t^\tau \sigma_t(u)\left[dZ^*(u) - \left(\frac{\nu_t(u)}{\sigma_t(u)}\right) du\right]\right) \\
& = (V_t + X_t - C_t) Exp\left(\int_t^\tau (\mu_t(u) - r(u) - \nu_t(u))du - \frac{1}{2}\int_t^\tau \sigma_t^2(u)du + \int_t^\tau \sigma_t(u) dZ^*(u)\right),
\end{aligned}
$$

which is a martingale if and only if $\nu_t(u) = \mu_t(u) - r(u)$. It is easy to check that the values of all derivative assets are also martingales under Q. $Q.E.D.$

Proof of Proposition 1: To preclude arbitrage, the market value at t of the asset payoff at $t+1$ must be equal to the investment cost $V_t + X_t - C_t + W_t(X_t)$, i.e.,

$$\begin{aligned} V_t + X_t - C_t + W_t(X_t) &= E_t^{Q^\nu}\left(\beta_t\left[V_{t+1}(V_t + X_t - C_t, \sigma_t) - M_{t+1}(V_{t+1})\right]\right) \\ \Longleftrightarrow \quad V_t + X_t - C_t &= (V_t + X_t - C_t)Exp\left(\int_t^{t+1}[\mu_t(u) - r(u) - \nu_t(u)]du\right) \\ &\quad - E_t^{Q^\nu}\left(\beta_t M_{t+1}(V_{t+1}) - W_t(X_t)\right), \end{aligned} \tag{A1}$$

which will be true if and only if $\nu_t < \mu_t(u) - r(u)$. $Q.E.D.$

The market price of debt in equilibrium (if one exists) is

$$\hat{d}(v_t) = (1 - \hat{b}(v_t))\text{min}(v_t, 1) + \hat{b}(v_t)\left[\hat{c}(v_t) + E_t^{Q^\nu}\beta_t\left(\hat{d}(v_{t+1}) - \alpha(v_{t+1})\right)\right], \tag{A2}$$

where, under martingale P, the liquidation value of assets at time $t+1$ is a function of equilibrium choices at t,

$$\begin{aligned} v_{t+1} &\equiv v_{t+1}(v_t + \hat{x}(v_t) - \hat{c}(v_t), \hat{\sigma}(v_t)) = \frac{1}{F}V_{t+1}(V_t + \hat{X}(V_t, F) - \hat{C}(V_t, F), \hat{\sigma}(V_t, F)) \\ &= (v_t + \hat{x}(v_t) - \hat{c}(v_t))Exp\left(\int_t^{t+1}\left(\mu_t(u) - \frac{1}{2}[\hat{\sigma}(v_t)]^2(u)\right)du + \int_t^{t+1}\hat{\sigma}(v_t)(u)dZ(u)\right) \end{aligned} \tag{A3}$$

We use $x \in \{x' | x' \geq -(v-1)^+\}$ and $c \in \{c' | c' \leq v_t + x\}$ to denote feasible values of the amount of equity and coupon rate per dollar of debt; σ to denote a feasible volatility process $\{\sigma(u) \in \Sigma,\ u \in [t, t+1]\}$, and $b \in \{0, 1\}$ to denote whether or not debtholders seek a bankruptcy of the firm. We continue to use Q^ν as an equivalent martingale measure defined by a feasible process $\{\nu(u),\ u \in [t, t+1]\}$, satisfying the integrability condition for risk-premia. To show that an equilibrium exists, we define, corresponding to each feasible triplet (x, σ, ν), an operator $\mathcal{T}(\cdot)$ which maps the set of all bounded, continuous functions, $h(\cdot)$, into the same set of functions:

$$\begin{aligned} \mathcal{T}h(v_t) = \underset{b \in \{0,1\}, c \leq v_t + x}{\text{Maximize}} \quad & (1-b)\text{min}(v_t, 1) + bG(v_t, x, c, \sigma, \nu) \\ \text{subject to} \quad & G(v_t, x, c, \sigma, \nu) \leq \text{min}(v_t, 1), \end{aligned} \tag{A4}$$

where

$$G(v_t, x, c, \sigma, \nu) \equiv c + E_t^{Q^\nu}\left[\beta_t \kappa(v_{t+1}(v_t + x - c, \sigma))\right], \tag{A5}$$

$$\kappa(v_{t+1}(v_t + x - c, \sigma)) \equiv h(v_{t+1}(v_t + x - c, \sigma)) - \alpha(v_{t+1}(v_t + x - c, \sigma)). \tag{A6}$$

Given a feasible triplet (x, σ, ν), there exists a continuous, unique and bounded $h(v_t) = \mathcal{T}h(v_t)$ for every feasible v_t if: (i) $\mathcal{T}h_1(v_t) \geq \mathcal{T}h_2(v_t)$ for any two bounded, continuous functions $h_1(v_t)$ and $h_2(v_t)$ with

$h_1(v_t) \geq h_2(v_t)$, (ii) $\mathcal{T}(h(v_t) + a) = \mathcal{T}h(v_t) + a\beta_t$ for any constant a and for any bounded, continuous function $h(v_t)$, and (iii) $\beta_t < 1$. [See, e.g., Blackwell (1965)]. Since the ''cost per-stage,'' $\min(1, v_t)$ is bounded, conditions (i)-(iii) hold. By using Proposition 2 in Section 7.1 of Bertsekas, it can be shown that the optimal solution $h(v_t)$ is indeed continuous, unique and bounded when $\beta_t < 1$. Further, if one such solution $h(v_t)$ can be explicitly derived then it is unique corresponding to (x, σ, ν). We derive such explicit solutions for each feasible (x, σ, ν). Thus, the $h(v_t)$ that corresponds to the triplet of (x, σ, ν), which is compatible with the shareholder objective, is the desired equilibrium market price of debt $\hat{d}(v_t)$. The solution c to (23) is then the equilibrium $\hat{c}(v_t)$.

Proof of Lemma 2: If the shareholders infuse new capital X_t, conditional on the firm's closure, their net equity value after recouping the cost of new capital, is $(V_t + X_t - F)^+ - X_t - W_t$. The net equity value when the firm is closed and no new capital is raised is equal to $(V_t - F)^+$. To see that $(V_t + X_t - F)^+ - X_t - W_t \leq (V_t - F)^+,\ \forall X_t \geq -(V_t - F)^+$, let $X_t = \epsilon - (V_t - F)^+$ for $\epsilon \geq 0$. Then for $V_t \leq F,\ X_t = \epsilon$, and

$$\begin{aligned} & (V_t + X_t - F)^+ - X_t - W_t \leq (V_t - F)^+ \\ \iff & (V_t + \epsilon - F)^+ - \epsilon - W_t \leq 0, \end{aligned}$$

which holds for all $\epsilon \geq 0$. If $V_t > F$, then $X_t = \epsilon - (V_t - F)$ and

$$\begin{aligned} & (V_t + X_t - F)^+ - X_t - W_t \leq (V_t - F)^+ \\ \iff & (\epsilon)^+ - \epsilon - W_t \leq 0, \end{aligned}$$

which also holds since $\epsilon \geq 0$. $Q.E.D.$

Proof of Proposition 2: Fix (x, σ, ν). Then suppose that (i) $G(v_t, x, c, \sigma, \nu)$ is increasing in c, given (v_t, x, σ, ν), and that (ii) $x = -(v_t - 1)^+ + E_t^{Q^\nu} \beta_t \alpha(0) + \epsilon,\ \epsilon \geq 0$ if and only if $b = 1$ and $x = 0$ otherwise. We then derive an h for (23) and show that suppositions (i)-(ii) hold for this h, thus proving that this h is the unique solution for (x, σ, ν).

By supposition (i), G attains its largest feasible value [corresponding to $c = v_t + x$] equal to $G(v_t, x, v_t + x, \sigma, \nu)$. Thus,

$$c = \begin{cases} v_t + x & \text{if } G(v_t, x, v_t + x, \sigma, \nu) < \min(v_t, 1) \\ c' \in \{c' | G(v_t, x, c', \sigma, \nu) = \min(v_t, 1)\} & \text{otherwise.} \end{cases} \tag{A7}$$

We can have an explicit solution $h(v_t) = \mathcal{T}h(v_t)$ for each feasible (x, σ, ν) if and only if $h(v_t)$ satisfies [using the constraint in (23) and (A1) in (23)]

$$\begin{aligned} h(v_t) &= \max\left[\min(v_t, 1), \ \ \min\left(\min(v_t, 1), G(v_t, x, v_t + x, \sigma, \nu)\right)\right] \\ &= \max\Big[\min(v_t, 1), \ \ \min\Big(\min(v_t, 1), \ \ v_t + x + E_t^{Q^\nu}\beta_t\left[h(0) - \alpha(0)\right]\Big)\Big]. \end{aligned} \tag{A8}$$

(A1)-(A2) imply that

$$b = \begin{cases} 1 & \iff \ \min(v_t, 1) \ \le \ G(v_t, x, v_t + x, \sigma, \nu) = v_t + x + E_t^{Q^\nu}\beta_t\left[h(0) - \alpha(0)\right]. \\ & \iff \ \min(v_t, 1) \ = \ G(v_t, x, c, \sigma, \nu), \\ 0 & \qquad\qquad\qquad\qquad \text{otherwise.} \end{cases} \tag{A9}$$

In (A2), set $v_t = 0$, when $x = E_t^{Q^\nu}\beta_t\alpha(0) + \epsilon$ for $b = 1$ by supposition (ii); resulting in $h(0) = 0$, which implies that

$$h(v_t) = \max\Big[\min(v_t, 1), \ \ \min\Big(\min(v_t, 1), \ v_t + x - E_t^{Q^\nu}\beta_t\alpha(0)\Big)\Big] = \min(v_t, 1), \tag{A10}$$

where the second equality follows by (A3). Thus, (A3) and (A4) imply (26) and (27). To verify supposition (i) that G is increasing in c, differentiate G with respect to c,

$$\begin{aligned} \frac{\partial G}{\partial c} &= 1 + E_t^{Q^\nu}\beta_t\left[\left(-\alpha'(v_{t+1}) + 1_{\{v_{t+1}<1\}}\right)\frac{\partial v_{t+1}}{\partial c}\right], \\ &= E_t^{Q^\nu}\beta_t\left[\frac{1}{\beta_t} + \alpha'(v_{t+1}) - 1_{\{v_{t+1}<1\}}\right], \\ &\ge 0, \end{aligned}$$

where the inequality follows from Assumption 2. To verify supposition (ii), observe that $x = 0$ if $b = 0$ by Lemma 2. (A3) implies that $b = 1$ if and only if $x \ge -(v_t - 1)^+ + E_t^{Q^\nu}\beta_t\alpha(0)$. $Q.E.D.$

Proof of Lemma 3: To prove this lemma, note that the undiscounted asset value $v_{t+1}(v + x - c, \sigma)$ under Q^ν, given (v_t, x, c, σ, ν), is:

$$v_{t+1} = (v_t + x - c)Exp\left(\int_t^{t+1}(\mu_t(u) - \nu(u))du - \frac{1}{2}\int_t^{t+1}\sigma_t^2(u)du + \int_t^{t+1}\sigma_t(u)dZ^*(u)\right), \tag{A11}$$

where the Brownian motion Z^* is defined in the proof of Lemma 1. Thus, under Q^ν, v_{t+1} is lognormally distributed, given (v_t, x, c, σ, ν). Further, observe that by (26)-(27), $\kappa(\cdot)$ in (25) is strictly concave and increasing. We need to prove this lemma for the admissible (x, σ, ν) for which $\{b = 1\}$.

(i) To prove that $\tilde{c}(v_t + x, \sigma, \nu)$ is non-increasing in x, let $x^1 > x^2$ with $(x^i, \sigma, \nu) \in \{b = 1\}$ for $i = 1, 2$. Then for $i = 1, 2$,

$$\begin{aligned} G(v_t, x^i, \tilde{c}(v_t + x^i, \sigma, \nu), \sigma, \nu) &= \tilde{c}(v_t + x^i, \sigma, \nu) + E_t^{Q^\nu}\left[\beta_t \kappa(v_{t+1}(v_t + x^i - \tilde{c}(v_t + x^i, \sigma, \nu), \sigma))\right] \\ &= \min(v_t, 1), \end{aligned} \tag{A12}$$

which implies that

$$\begin{aligned} G(v_t, x^1, \tilde{c}(v_t + x^1, \sigma, \nu), \sigma, \nu) &= \min(v_t, 1) \\ &= G(v_t, x^2, \tilde{c}(v_t + x^2, \sigma, \nu), \sigma, \nu) \\ &= \tilde{c}(v_t + x^2, \sigma, \nu) + E_t^{Q^\nu}\left[\beta_t \kappa(v_{t+1}(v_t + x^2 - \tilde{c}(v_t + x^2, \sigma, \nu), \sigma))\right] \\ &\leq \tilde{c}(v_t + x^2, \sigma, \nu) + E_t^{Q^\nu}\left[\beta_t \kappa(v_{t+1}(v_t + x^1 - \tilde{c}(v_t + x^2, \sigma, \nu), \sigma))\right] \\ &= G(v_t, x^1, \tilde{c}(v_t + x^2, \sigma, \nu), \sigma, \nu), \end{aligned}$$

where the inequality follows since, under Q^ν, the distribution of $v_{t+1}(v_t + x^1 - \tilde{c}(v_t + x^2, \sigma, \nu), \sigma)$, given $(v_t + x^1, \tilde{c}(v_t + x^2, \sigma, \nu), \sigma, \nu)$, first order stochastically dominates the distribution of $v_{t+1}(v_t + x^2 - \tilde{c}(v_t + x^2, \sigma, \nu), \sigma)$, given $(v_t + x^2, \tilde{c}(v_t + x^2, \sigma, \nu), \sigma, \nu)$, and $\kappa(\cdot)$ is concave and increasing. Since $G(v_t, x, c, \sigma, \nu)$ is a non-decreasing function of c, given (v_t, x, σ, ν), $\tilde{c}(v_t + x^1, \sigma, \nu) \leq \tilde{c}(v_t + x^2, \sigma, \nu)$. $Q.E.D.$

To prove that $\tilde{c}(v_t + x, \sigma, \nu)$ is convex in x, given (v_t, σ, ν), choose $(x^i, \sigma, \nu) \in \{b = 1\}$ for $i = 0, 1$. For $\lambda \in (0, 1)$, let $x^\lambda = \lambda x^0 + (1 - \lambda) x^1$. Then $(x^\lambda, \sigma, \nu) \in \{b = 1\}$ by (27). It follows that

$$\begin{aligned} \min(v_t, 1) &= G(v_t, x^\lambda, \tilde{c}(v_t + x^\lambda, \sigma, \nu), \sigma, \nu) \\ &= \lambda G(v_t, x^0, \tilde{c}(v_t + x^0, \sigma, \nu), \sigma, \nu) + (1 - \lambda) G(v_t, x^1, \tilde{c}(v_t + x^1, \sigma, \nu), \sigma, \nu) \\ &= \lambda \left\{\tilde{c}(v_t + x^0, \sigma, \nu) + E_t^{Q^\nu}\left[\beta_t \kappa(v_{t+1}(v_t + x^0 - \tilde{c}(v_t + x^0, \sigma, \nu), \sigma))\right]\right\} \\ &\quad + (1 - \lambda) \left\{\tilde{c}(v_t + x^1, \sigma, \nu) + E_t^{Q^\nu}\left[\beta_t \kappa(v_{t+1}(v_t + x^1 - \tilde{c}(v_t + x^1, \sigma, \nu), \sigma))\right]\right\} \\ &\leq \lambda \tilde{c}(v_t + x^0, \sigma, \nu) + (1 - \lambda) \tilde{c}(v_t + x^1, \sigma, \nu) \\ &\quad + E_t^{Q^\nu}\Big[\beta_t \kappa(\lambda v_{t+1}(v_t + x^0 - \tilde{c}(v_t + x^0, \sigma, \nu), \sigma) \end{aligned}$$

$$
\begin{aligned}
&\qquad +(1-\lambda)v_{t+1}(v_t + x^1 - \tilde{c}(v_t + x^1, \sigma, \nu), \sigma))\Big] \\
&= \lambda \tilde{c}(v_t + x^0, \sigma, \nu) + (1-\lambda)\tilde{c}(v_t + x^1, \sigma, \nu) \\
&\qquad + E_t^{Q^\nu}\Big[\beta_t \kappa \Big(v_{t+1}\Big(v_t + x^\lambda - \lambda \tilde{c}(v_t + x^0, \sigma, \nu) - (1-\lambda)\tilde{c}(v_t + x^1, \sigma, \nu), \sigma\Big)\Big)\Big] \\
&= G(v_t, x^\lambda, \lambda \tilde{c}(v_t + x^0, \sigma, \nu) + (1-\lambda)\tilde{c}(v_t + x^1, \sigma, \nu), \sigma, \nu),
\end{aligned}
$$

where the inequality follows since $\kappa(\cdot)$ is concave. As $G(v_t, x, c, \sigma, \nu)$ is non-decreasing in c, given (v_t, x, σ, ν),

$$
\lambda \tilde{c}(v_t + x^0, \sigma, \nu) + (1-\lambda)\tilde{c}(v_t + x^1, \sigma, \nu) \geq \tilde{c}(v_t + x^\lambda, \sigma, \nu), \tag{A13}
$$

which proves that $\tilde{c}(v_t + x, \sigma, \nu)$ is convex in x, given (v_t, x, σ). $Q.E.D.$

(ii) To prove that $\tilde{c}(v_t + x, \sigma, \nu)$ non-decreases in σ, given (v_t, x, ν), let $\sigma^1 > \sigma^2$ and $(x, \sigma^i, \nu) \in \{b = 1\}$ for $i = 1, 2$. Then

$$
\begin{aligned}
G(v_t, x, \tilde{c}(v_t + x, \sigma^1, \nu), \sigma^1, \nu) &= \min(v_t, 1) \\
&= G(v_t, x, \tilde{c}(v_t + x, \sigma^2, \nu), \sigma^2, \nu) \\
&\geq G(v_t, x, \tilde{c}(v_t + x, \sigma^2, \nu), \sigma^1, \nu),
\end{aligned} \tag{A14}
$$

where the inequality follows since

$$
E_t^{Q^\nu}\Big[\beta_t \kappa(v_{t+1}(v_t + x - c, \sigma^1))\Big] \leq E_t^{Q^\nu}\Big[\beta_t \kappa(v_{t+1}(v_t + x - c, \sigma^2))\Big], \ \text{ for } \sigma^1 > \sigma^2, \tag{A15}
$$

when the distribution under Q^ν of $v_{t+1}(v_t + x - c, \sigma^1)$ second-order stochastically dominates the distribution under Q^ν of $v_{t+1}(v_t + x - c, \sigma^2)$, given (v_t, x, c, ν), and $\kappa(\cdot)$ is concave. Since $G(v_t, x, c, \sigma^1, \nu)$ is increasing in c, given (v_t, x, σ^1, ν), (A8) implies that

$$
\tilde{c}(v_t + x, \sigma^1, \nu) \geq \tilde{c}(v_t + x, \sigma^2, \nu), \tag{A16}
$$

which proves that $\tilde{c}(v_t + x, \sigma, \nu)$ is increasing in σ, given (v_t, x, ν). $Q.E.D.$

(iii) To prove that $\tilde{c}(v_t + x, \sigma, \nu)$ is non-decreasing in ν, suppose that $\nu^1 > \nu^2$ and $(x, \sigma, \nu^i) \in \{b = 1\}$. Let $Q^{\nu i}$ be equivalent martingale measure, corresponding to risk premium ν^i, $i = 1, 2$. By (A5) the distribution under $Q^{\nu 2}$ of v_{t+1} first-order stochastically dominates the distribution under $Q^{\nu 1}$ of v_{t+1}, given (v_t, x, c, σ), which implies that

$$E_t^{Q^{\nu 2}}\left[\beta_t \kappa(v_{t+1}(v_t + x - c, \sigma))\right] \geq E_t^{Q^{\nu 1}}\left[\beta_t \kappa(v_{t+1}(v_t + x - c, \sigma))\right]. \tag{A17}$$

Then, since $(x, \sigma, \nu^i) \in \{b = 1\}$,

$$\begin{aligned} G(v_t, x, \tilde{c}(v_t + x, \sigma, \nu^1), \sigma, \nu^1) &= \min(v_t, 1) \\ &= G(v_t, x, \tilde{c}(v_t + x, \sigma, \nu^2), \sigma, \nu^2) \\ &\geq G(v_t, x, \tilde{c}(v_t + x, \sigma, \nu^2), \sigma, \nu^1), \end{aligned} \tag{A18}$$

where the inequality follows from (A11). Since $G(v_t, x, c, \sigma, \nu^1)$ is increasing in c, given (v_t, x, σ, ν^1), (A12) implies that

$$\tilde{c}(v_t + x, \sigma, \nu^1) \geq \tilde{c}(v_t + x, \sigma, \nu^2), \tag{A19}$$

which proves that $\tilde{c}(v_t + x, \sigma, \nu)$ is non-decreasing in ν, given (v_t, x, σ). $Q.E.D.$

Proof of Lemma 4: (i) $\tilde{c}(v_t + x, \sigma, \nu)$ is a solution c to $c + E_t^{Q^\nu}\left[\beta_t\left(\min(v_{t+1}, 1) - \alpha(v_{t+1})\right)\right] = \min(v_t, 1)$, which implies [letting $v_{t+1} \equiv v_{t+1}(v_t + x + c, \sigma)$] that

$$E_t^{Q^\nu}\left[\beta_t \min(v_{t+1}, 1)\right] = E_t^{Q^\nu}\left[\beta_t \alpha(v_{t+1})\right] + \min(v_t, 1) - c. \tag{A20}$$

Then $f(v_t, x, c, \sigma, \nu)$ in (29) is given by

$$\begin{aligned} f(v_t, x, c, \sigma, \nu) &= E_t^{Q^\nu}\left[\beta_t\left(v_{t+1} - 1\right)^+\right] - x - w(v_t)(x)^+ \\ &= E_t^{Q^\nu}\left[\beta_t v_{t+1} - \beta_t \min(v_{t+1}, 1)\right] - x - w(v_t)(x)^+ \\ &= E_t^{Q^\nu}\left[\beta_t v_{t+1}\right] - E_t^{Q^\nu}\left[\beta_t \alpha(v_{t+1})\right] - \min(v_t, 1) + c - x - w(v_t)(x)^+ \\ &= (v_t + x - c)\gamma_t - E_t^{Q^\nu}\left[\beta_t \alpha(v_{t+1})\right] - \min(v_t, 1) + c - x - w(v_t)(x)^+ \\ &= (v_t + x - c)(\gamma_t - 1) - E_t^{Q^\nu}\left[\beta_t \alpha(v_{t+1})\right] - w(v_t)(x)^+ + (v_t - 1)^+, \end{aligned}$$

where the third equality follows from (A14) and γ_t is given by

$$\gamma_t \equiv Exp\left(\int_t^{t+1} [\mu_t(u) - r(u) - \nu(u)] du\right). \tag{A21}$$

(ii) By partially differentiating $f(v_t, x, c, \sigma, \nu)$ with respect to c

$$\begin{aligned} f_c \equiv \frac{\partial f(v_t, x, c, \sigma, \nu)}{\partial c} &= -(\gamma_t - 1) - \frac{\partial}{\partial c} E_t^{Q^\nu} \left[\beta_t \alpha(v_{t+1})\right] \\ &= -(\gamma_t - 1) - E_t^{Q^\nu} \left[\beta_t \alpha'(v_{t+1}) \frac{\partial v_{t+1}}{\partial c}\right] \leq 0, \end{aligned} \quad \text{(A22)}$$

where the inequality follows since $\alpha' < 0$ by Assumption 2 and $\partial v_{t+1}/\partial c = -1$.

Since $\alpha(v_{t+1})$ is convex and decreasing and the distribution under Q^ν of $v_{t+1}(v_t + x - c, \sigma^2)$ second-order stochastically dominates the distribution under Q^ν of $v_{t+1}(v_t + x - c, \sigma^1)$, given (v_t, x, c, ν) and $\sigma^1 > \sigma^2$,

$$E_t^{Q^\nu} \left[\beta_t \alpha(v_{t+1}(v_t + x - c, \sigma^1)\right] \geq E_t^{Q^\nu} \left[\beta_t \alpha(v_{t+1}(v_t + x - c, \sigma^2)\right], \quad \text{(A23)}$$

which implies that

$$f_\sigma \equiv \frac{\partial}{\partial \sigma} f(v_t, x, c, \sigma, \nu) < 0. \quad \text{(A24)}$$

Let $\nu^1 > \nu^2$ and $Q^{\nu i}$ be the equivalent pricing measure corresponding to ν^i, $i = 1, 2$. Then the distribution under $Q^{\nu 2}$ of $v_{t+1}(v_t + x - c, \sigma)$ first-order stochastically dominates the distribution under $Q^{\nu 1}$ of $v_{t+1}(v_t + x - c, \sigma)$, given (v_t, x, c, σ), which implies for a convex decreasing $\alpha(\cdot)$ that

$$E_t^{Q^{\nu 1}} \left[\beta_t \alpha(v_{t+1}(v_t + x - c, \sigma)\right] \geq E_t^{Q^{\nu 2}} \left[\beta_t \alpha(v_{t+1}(v_t + x - c, \sigma)\right]. \quad \text{(A25)}$$

Further, $\partial \gamma_t / \partial \nu < 0$, which implies

$$f_\nu \equiv \frac{\partial}{\partial \nu} f(v_t, x, c, \sigma, \nu) < 0. \quad \text{(A26)}$$

(iii) This easily follows. $Q.E.D.$

Proof of Proposition 3:

(i) Given (v_t, σ, ν), $f(v_t, x, \tilde{c}(v_t + x, \sigma, \nu), \sigma, \nu)$ is strictly concave in x. This is because $x - \tilde{c}(v_t + x, \sigma, \nu)$ is concave in x as $\tilde{c}(v_t + x, \sigma, \nu)$ is convex in x by Lemma 3-(i); $-\alpha(v_{t+1})$ is strictly concave and increasing in v_{t+1}, which is increasing in $x - \tilde{c}(v_t + x, \sigma, \nu)$; and $-w(v_t)(x)^+$ is non-increasing [decreasing over $x > 0$] and concave in x. The result thus follows, with the first-order condition for maximization,

$$f_x + f_c \tilde{c}_x = 0. \quad \text{(A27)}$$

$Q.E.D.$

(ii) Partially differentiating $f(v_t, x, \tilde{c}(v_t + x, \sigma, \nu), \sigma, \nu)$ with respect to σ,

$$\frac{\partial}{\partial \sigma} f(v_t, x, \tilde{c}(v_t + x, \sigma, \nu), \sigma, \nu) = f_c \tilde{c}_\sigma + f_\sigma < 0, \tag{A28}$$

where $\tilde{c}_j$ is the partial derivative of $\tilde{c}(v_t + x, \sigma, \nu)$ with respect to argument j, and the inequality follows since, given (v_t, x, ν), $\tilde{c}(v_t + x, \sigma, \nu)$ is increasing in σ by Lemma 3-(ii), and $f_c < 0$, $f_\sigma < 0$ by Lemma 4-(ii). $Q.E.D.$

(iii) Partially differentiating $f(v_t, x, \tilde{c}(v_t + x, \sigma, \nu), \sigma, \nu)$ with respect to ν,

$$\frac{\partial}{\partial \nu} f(v_t, x, \tilde{c}(v_t + x, \sigma, \nu), \sigma, \nu) = f_c \tilde{c}_\nu + f_\nu < 0, \tag{A29}$$

where the inequality follows since, given (v_t, x, σ), $\tilde{c}(v_t + x, \sigma, \nu)$ is increasing in ν by Lemma 3-(ii), and $f_c < 0$ and $f_\nu < 0$ by Lemma 5-(ii). $Q.E.D.$

(iv) This easily follows. $Q.E.D.$

(v) Partially differentiating $f(v_t, \tilde{x}(v_t, \sigma, \nu), \tilde{c}(v_t + \tilde{x}(v_t, \sigma, \nu), \sigma, \nu), \sigma, \nu)$ with respect to σ,

$$\begin{aligned} \frac{\partial}{\partial \sigma} f(v_t, \tilde{x}(v_t, \sigma, \nu), \tilde{c}(v_t + \tilde{x}(v_t, \sigma, \nu), \sigma, \nu), \sigma, \nu) &= f_x \tilde{x}_\sigma + f_c[\tilde{c}_x \tilde{x}_\sigma + \tilde{c}_\sigma] + f_\sigma \\ &= \tilde{x}_\sigma [f_x + f_c \tilde{c}_x] + f_c \tilde{c}_\sigma + f_\sigma \\ &= f_c \tilde{c}_\sigma + f_\sigma < 0, \end{aligned} \tag{A30}$$

where the third equality follows from (A21) and the inequality follows from (A22). $Q.E.D.$

(vi) Partially differentiating $f(v_t, \tilde{x}(v_t, \sigma, \nu), \tilde{c}(v_t + \tilde{x}(v_t, \sigma, \nu), \sigma, \nu), \sigma, \nu)$ with respect to ν,

$$\begin{aligned} \frac{\partial}{\partial \nu} f(v_t, \tilde{x}(v_t, \sigma, \nu), \tilde{c}(v_t + \tilde{x}(v_t, \sigma, \nu), \sigma, \nu), \sigma, \nu) &= f_x \tilde{x}_\nu + f_c[\tilde{c}_x \tilde{x}_\nu + \tilde{c}_\nu] + f_\nu \\ &= \tilde{x}_\nu [f_x + f_c \tilde{c}_x] + f_c \tilde{c}_\nu + f_\nu \\ &= f_c \tilde{c}_\nu + f_\nu < 0, \end{aligned} \tag{A31}$$

where the third equality follows from (A21) and the inequality follows from (A23). $Q.E.D.$

(vii) Let $v_t^1 > v_t^2 > 1$. Then

$$\begin{aligned} f\left(v_t^2, \tilde{x}(v_t^2, \sigma, \nu), \tilde{c}(v_t^2 + \tilde{x}(v_t^2, \sigma, \nu), \sigma, \nu), \sigma, \nu\right) &< f\left(v_t^1, \tilde{x}(v_t^2, \sigma, \nu), \tilde{c}(v_t^2 + \tilde{x}(v_t^2, \sigma, \nu), \sigma, \nu), \sigma, \nu\right) \\ &< f\left(v_t^1, \tilde{x}(v_t^2, \sigma, \nu), \tilde{c}(v_t^1 + \tilde{x}(v_t^2, \sigma, \nu), \sigma, \nu), \sigma, \nu\right) \\ &\leq f\left(v_t^1, \tilde{x}(v_t^1, \sigma, \nu), \tilde{c}(v_t^1 + \tilde{x}(v_t^1, \sigma, \nu), \sigma, \nu), \sigma, \nu\right), \end{aligned}$$

where the first inequality follows since $f(v_t, x, c, \sigma, \nu)$ is increasing in v_t, given (x, c, σ, ν); the second inequality follows since $\tilde{c}(v_t + x, \sigma, \nu)$ is decreasing in v_t, given (x, σ, ν) [by a proof similar to Lemma 3-(i)], and $f(v_t, x, c, \sigma, \nu)$ is decreasing in c, given (v_t, x, σ, ν); and the third inequality follows since $\tilde{x}(v_t^1, \sigma, \nu)$ is the maximizer of $f(v_t, x, \tilde{c}(v_t + x, \sigma, \nu), \sigma, \nu)$ with respect to x, given $v_t = v_t^1$ and (σ, ν). $Q.E.D.$

(viii) Define

$$\tilde{f}(v_t, \sigma, \nu) \equiv f(v_t, \tilde{x}(v_t, \sigma, \nu), \tilde{c}(v_t + \tilde{x}(v_t, \sigma, \nu), \sigma, \nu), \sigma, \nu) - (v_t - 1)^+. \tag{A32}$$

$\tilde{f}(v_t, \sigma, \nu)$ is monotonically decreasing in σ, given (v_t, ν), by Proposition 3-(v), and monotonically decreasing in ν, given (v_t, σ_t) by Proposition 3-(vi). Thus, there exists $\hat{v}(0)$ as defined in the text such that for all $v_t < \hat{v}(0)$, $\tilde{f}(v_t, \underline{\sigma}, 0) < 0$ and there exist no $\sigma \in \Sigma$ and $\nu \in (0, \mu_t - r)$ which will solve $\tilde{f}(v_t, \sigma, \nu) = 0$. Further, for no finite v_t, there will exist (σ, ν) that will solve $\tilde{f}(v_t, \sigma, \nu) = 0$ if $\hat{v}(0) = \infty$, when the firm will not come into existence.

Now, suppose that $\hat{v}(0) < \infty$. Then, for every feasible $\nu \in (0, \mu_t - r)$, there exists $\hat{v}(\nu)$ as defined in the text such that, given (v_t, ν), there exists no $\sigma \in \Sigma$, which will solve $\tilde{f}(v_t, \sigma, \nu) = 0$ if $v_t < \hat{v}(\nu)$ for which $\tilde{f}(v_t, \underline{\sigma}, \nu) < 0$. Since $\tilde{f}(v_t, \sigma, \nu)$ is increasing in v_t by Proposition 3-(vii), $\tilde{f}(v_t, \sigma, \nu) < 0$, for all $v_t < \hat{v}(\nu)$ and $\sigma \in \Sigma$. Thus, given (v_t, ν), $bl = 0$ if and only if $v_t < \hat{v}(\nu)$. If $v_t \geq \hat{v}(\nu)$, then, $\tilde{f}(v_t, \underline{\sigma}, \nu) \geq 0$, and since $\tilde{f}(v_t, \sigma, \nu)$ is decreasing σ, there will exist (by continuity) a $\sigma > \underline{\sigma}$ such that $\tilde{f}(v_t, \sigma, \nu) = 0$ if $\lim_{\sigma \to \overline{\sigma}} \tilde{f}(v_t, \sigma, \nu) \leq 0$. Observe that

$$\begin{aligned} \lim_{\sigma \to \overline{\sigma}} \tilde{f}(v_t, \sigma, \nu) &= (v_t + \tilde{x}(v_t, \overline{\sigma}, \nu) - \tilde{c}(v_t + \tilde{x}(v_t, \overline{\sigma}, \nu), \overline{\sigma}, \nu))(\gamma_t - 1) \\ &\quad - \lim_{\sigma \to \overline{\sigma}} E_t^{Q^\nu} \beta_t \alpha(v_{t+1}) - w(v_t)(\tilde{x}(v_t, \overline{\sigma}, \nu))^+ \\ &= (v_t + \tilde{x}(v_t, \overline{\sigma}, \nu) - \tilde{c}(v_t + \tilde{x}(v_t, \overline{\sigma}, \nu), \overline{\sigma}, \nu))(\gamma_t - 1) \\ &\quad - \beta_t \bar{\alpha} - w(v_t)(\tilde{x}(v_t, \overline{\sigma}, \nu))^+ \leq 0, \end{aligned} \tag{A33}$$

where the equality follows by Assumption 2 and the inequality by Assumption 1 [divide by F and use the definition $F\bar{\alpha} = \bar{M}$.

(ix) Now, denote the σ which solves $\tilde{f}(v_t, \sigma, \nu) = 0$, by $\tilde{\sigma}(v_t, \nu)$. Thus, if $\nu = \hat{\nu}(v_t)$ such that

$v_t \geq \hat{v}(\hat{\nu}(v_t))$, then $\hat{\sigma}(v_t) = \tilde{\sigma}(v_t, \hat{\nu}(v_t))$. Further, $\hat{x}(v_t) = \tilde{x}(v_t, \hat{\sigma}(v_t), \hat{\nu}(v_t))$ and $\hat{c}(v_t) = \tilde{c}(v_t + \hat{x}(v_t), \hat{\sigma}(v_t), \hat{\nu}(v_t))$ by the definition of $\hat{v}$. $Q.E.D.$

Proof of Proposition 4: (i) Consider $\nu^1 > \nu^2$ and choose $v_t \geq \hat{v}(\nu^1)$ and $v_t \geq \hat{v}(\nu^2)$. Given this asset/debt ratio, the firm will remain open until at least $t+1$, whether the risk premium is ν^1 or ν^2. Then there exist shareholders' choices $(\tilde{x}(v_t, \sigma^i, \nu^i), \sigma^i)$, corresponding to risk premium $\nu^i,\ i = 1, 2$, in equilibrium. We have to show that $\sigma^1 < \sigma^2$. Since $(\tilde{x}(v_t, \sigma^i, \nu^i), \sigma^i)$ is in equilibrium, for $i = 1, 2$,

$$\tilde{f}(v_t, \sigma^i, \nu^i) = f\left(v_t, \tilde{x}\left(v_t, \sigma^i, \nu^i\right), \tilde{c}\left(v_t, \tilde{x}(v_t, \sigma^i, \nu^i), \sigma^i, \nu^i\right), \sigma^i, \nu^i\right) - (v_t - 1)^+ = 0. \quad \text{(A34)}$$

Observe that

$$\tilde{f}\left(v_t, \sigma^1, \nu^2\right) > \tilde{f}\left(v_t, \sigma^1, \nu^1\right) = \tilde{f}\left(v_t, \sigma^2, \nu^2\right) = 0, \quad \text{(A35)}$$

where the inequality follows since $\tilde{f}(v_t, \sigma, \nu)$ is decreasing in ν, given (v_t, σ) by Proposition 3-(vi) and the equalities hold by (A28). (A29) implies that $\sigma^1 < \sigma^2$, because $\tilde{f}(v_t, \sigma, \nu)$ is decreasing in σ, given (v_t, ν) by Proposition 3-(v). $Q.E.D.$

(ii) Fix ν and consider two values of asset/debt ratio, $v_t^1 > v_t^2 > \hat{v}(\nu)$. Given either asset/debt ratio, the firm will remain open until at least $t+1$. Then there exist shareholders' choices $(\tilde{x}(v_t^1, \sigma^1, \nu), \sigma^1)$ and $(\tilde{x}(v_t^2, \sigma^2, \nu), \sigma^2)$, corresponding to v_t^1 and v_t^2, respectively, which implies that, for $i = 1, 2$,

$$\tilde{f}(v_t^i, \sigma^i, \nu) = f\left(v_t^i, \tilde{x}(v_t^i, \sigma^i, \nu), \tilde{c}(v_t^i + \tilde{x}(v_t^i, \sigma^i, \nu), \sigma^i, \nu), \sigma^i, \nu\right) - (v_t^i - 1)^+ = 0. \quad \text{(A36)}$$

To show that $\sigma^1 > \sigma^2$, observe that

$$\tilde{f}\left(v_t^2, \sigma^1, \nu\right) < \tilde{f}\left(v_t^1, \sigma^1, \nu\right) = \tilde{f}\left(v_t^2, \sigma^2, \nu\right) = 0, \quad \text{(A37)}$$

where the inequality follows since $\tilde{f}(v_t, \sigma, \nu)$ is increasing in v_t, given (σ, ν), by Proposition 3-(vii); the equality holds by (A30). (A31) implies that $\sigma^1 > \sigma^2$, because $\tilde{f}(v_t, \sigma, \nu)$ is increasing in σ, given (v_t, ν) by Proposition 3-(v). $Q.E.D.$

Proofs of (iii) and (v) are similar to (i) and (ii).

(v) Let $\nu^1 > \nu^2$. Then

$$\tilde{f}(v, \underline{\sigma}, \nu^1) < \tilde{f}(v, \underline{\sigma}, \nu^2), \ \ \forall v \geq 0, \quad \text{(A38)}$$

since $\tilde{f}(v, \underline{\sigma}, \nu)$ is decreasing in ν. Thus $\hat{v}(\nu^1) > \hat{v}(\nu^2)$. $Q.E.D.$

For market valuation, a probability measure Q on $(\Omega, \mathcal{F})$ is called an equivalent martingale (pricing) measure if (a) Q is equivalent to P in the sense that $P(A) > 0$ if and only if $Q(A) > 0$ for any $A \in \Omega$, i.e., a change of measure density, $\xi_t(u)$, defined by

$$\xi_t(u) = E_u^P\left(\frac{dQ}{dP}\right), \;\; u \in [t, t+1]. \tag{A39}$$

exists almost surely, where $E_u^k(\cdot)$ is the conditional expectation under measure $k = P, Q$, given $\mathcal{F}_u$; and (b) the discounted value of an investment, $\{\tilde{\beta}_t(u)I_t(u),\; u \in [t, t+1]\}$, (in the assets, equity or debt) is a martingale under Q [equivalently, $\{\tilde{\beta}_t(u)I_t(u)\xi_t(u),\; u \in [t, t+1]\}$ is a martingale under P], implying that

$$I_t(t) = E_t^Q\left(\beta_t I_t(t+1)\right) = E_t^P\left(\beta_t I_t(t+1)\xi_t(t+1)\right). \tag{A40}$$

A pricing measure Q exists if and only if ξ_t exists. Arbitrage opportunities are absent if and only if a pricing measure exists, implying that no nonpositive investment results in a positive future value, i.e., $\nexists\, I_t(t) \leq 0$ with $I_t(\tau) > 0,\; \tau \in [t, t+1]$. Then, $\tilde{\beta}_t(u)\xi_t(u, \omega)$ is interpreted as the Arrow-Debreu state price at time t per unit of probability for one dollar to be delivered in state $\omega \in \Omega$ and time $u \in [t, t+1]$. We will call ξ_t as the state price, without using terms discounting and per unit probability. The following lemma shows the form of ξ_t.

Lemma 1: (a) The form of the state price (change of measure density) is given by

$$\xi_t(\tau) = \xi_t(t)Exp\left(-\int_t^\tau \left(\frac{\nu_t(u)}{\sigma_t(u)}\right) dZ(u) - \frac{1}{2}\int_t^\tau \left(\frac{\nu_t(u)}{\sigma_t(u)}\right)^2 du\right), \tag{A41}$$

with $\xi_t(t) = 1$. (b) If $M_t = W_t = 0$, then the asset values, $\{V_t(u)\}$, will be market values if and only if $\nu_t(u) = \mu_t(u) - r(u)$.

Proof: (a) The proof is available in standard text books. [See, e.g., Fleming and Rischel (1975).] (b) It can be easily checked that $\nu_t(u) = \mu_t(u) - r(u)$ if and only if $\{\tilde{\beta}_t(\tau)\tilde{V}_t(\tau)\xi_t(\tau),\; \tau \in [t, t+1]\}$ is a P-martingale, or $\{\tilde{\beta}_t(\tau)\tilde{V}_t(\tau),\; \tau \in [t, t+1]\}$ is a Q-martingale. Further, the values of all derivative assets are martingales under Q. [For an outline of the proof, see the Appendix.] $Q.E.D.$